AD-A083 336 TRW DEFENSE AND SPACE SYSTEMS GROUP REDONDO BEACH CA F/G 16/3
COST/BENEFIT ANALYSIS OF REENTRY VEHICLE HARDNESS TESTING USING--ETC(U)
MAR 79 L H DONAHUE, J J FARRELL DNA001-78-C-0135
UNCLASSIFIED TRW-33030-6003-RU-81-VOL- DNA-4949F-2 NL

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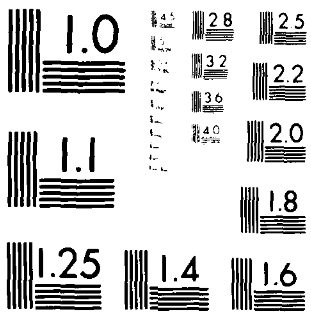

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COST/BENEFIT ANALYSIS OF REENTRY VEHICLE HARDNESS TESTING USING THE FAST METHODOLOGY

Volume II—Methods

TRW Defense & Space Systems Group
One Space Park
Redondo Beach, California 90278

30 March 1979

Final Report for Period 23 January 1978—30 March 1979

CONTRACT No. DNA 001-78-C-0135

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SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER
DNA 4949F-2

2. GOVT ACCESSION NO.
AD-A083 336

3. RECIPIENT'S CATALOG NUMBER

4. TITLE (and Subtitle)
COST/BENEFIT ANALYSIS OF REENTRY VEHICLE HARDNESS TESTING USING THE FAST METHODOLOGY
Volume II—Methods

5. TYPE OF REPORT & PERIOD COVERED
Final Report for Period
23 Jan 78—30 Mar 79

6. PERFORMING ORG. REPORT NUMBER
33030-6003-RU-81

7. AUTHOR(s)
L. H. Donahue
J. J. Farrell

8. CONTRACT OR GRANT NUMBER(s)
DNA 001-78-C-0135 new

9. PERFORMING ORGANIZATION NAME AND ADDRESS
TRW Defense and Space Systems Group
One Space Park
Redondo Beach, California 90278

10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
Subtask N99QAXAH116-06

11. CONTROLLING OFFICE NAME AND ADDRESS
Director
Defense Nuclear Agency
Washington, D.C. 20305

12. REPORT DATE
30 March 1979

13. NUMBER OF PAGES
64

14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)

15. SECURITY CLASS (of this report)
UNCLASSIFIED

15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

This work sponsored by the Defense Nuclear Agency under RDT&E RMSS Code B342078464 N99QAXAH11606 H2590D.

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Reentry Vehicles
Hardness Assessment
Test Data Analysis
Fragility Response Modeling
Maximum Likelihood Estimation
Conditional Probability
Non-Central t-Distribution

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The present study is an extension of previously reported Nuclear Hardness Evaluation Procedures (NHEP) efforts. In the current study, the Failure Analysis by Statistical Techniques (FAST) methodology was extended to provide new techniques for estimating the potential benefits of testing under uncertainty. A new computer code named ELF, Evaluation of Likelihood Function, was developed to provide a thorough evaluation of test results.

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UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

20. ABSTRACT (Continued)

To demonstrate the ELF code, this code was used to analyze magnetic flyer plate data on graphite resin material samples. Volume 1 of the Final Report, Applications, describes the results of the evaluations performed on the graphite resin magnetic flyer plate tests and their impact on predictions of reentry system performance. Also included in Volume 1 are the results of the analyses performed on a Lockheed Missiles and Space Company reference reentry vehicle. Volume 2 of the Final Report, Methods, describes in detail the development of the statistical and analytical techniques that were incorporated into the ELF code. Also reported in this volume is a new version of the FAST code, FAST7, Conditional Probability FAST, which shows great promise for test planning and evaluation analysis.

SUMMARY

The purpose of the study reported here was to develop and explore new techniques and procedures to aid in the design and assessment of reentry vehicles for survival against X-rays from exoatmospheric nuclear bursts. At present, a primary source of design information for reentry vehicle hardness against X-rays is underground nuclear testing. With the advent of a comprehensive nuclear test ban treaty, this source of proof data will no longer be available.

To prepare for the day when underground nuclear test data would no longer be available, Defense Nuclear Agency (DNA) initiated a program to develop a Nuclear Hardness Evaluation Procedure (NHEP). From this program has evolved the NHEP methodology, developed by Effects Technology, Inc., and a consortium of cooperating Agencies and Contractors. This methodology consists of a step-by-step general procedure for assessing reentry vehicle hardness, supported by a nuclear weapons effects data base.

An important tool which has been utilized to demonstrate the effectiveness of the NHEP methodology is the TRW-developed Failure Analysis by Statistical Techniques (FAST) code. The FAST code calculates the system probability of survival and associated confidence level for a system subjected to a defined environment. During two previous studies. TRW developed the use of FAST as a final step in the NHEP methodology.

In addition, new design optimization techniques were developed to identify optimal vehicle design configurations to assure maximum system hardness or minimum system weight at a prescribed probability of survival and confidence level. These optimization techniques were incorporated into a new computer code, BASH (Balanced System Hardness), and the utility of these design optimization techniques was verified by subsequent review of the BASH optimized design configurations, both by FAST code assessment and by engineering analysis.

The present study is an extension of those previously reported NHEP efforts. In the current study, the FAST methodology was extended to provide new techniques for estimating the potential benefits of testing under uncertainty. A new computer code named ELF, Evaluation of Likelihood Function, was developed to provide a thorough evaluation of test results. To provide a demonstration of the ELF code, this code was used to assess the graphite resin magnetic flyer plate test data.

A conditional probability version of the FAST code, FAST7, was developed which shows great promise for test planning and evaluation. In addition, as a part of this contractual effort, TRW provided consultation and support to Lockheed Missiles and Space Company (LMSC) in the FAST assessment of a Reference reentry vehicle design.

PREFACE

This final report documents the work completed by TRW Defense and Space Systems Group under the sponsorship of the Defense Nuclear Agency Contract DNA001-78-C-0135, entitled "Cost/Benefit Analysis of Reentry Vehicle Hardness Testing Using the FAST Methodology". The overall period of technical performance of this project was from 23 January 1978 to 30 March 1979. The TRW Project Manager for this work was Dr. John J. Farrell, replaced by Mr. Leo H. Donahue in the closing months of the project. The study was conducted within the Ground Systems and Technology Department, under the direction of Dr. Peter K. Dai. The authors are indebted to Dr. Charles Stein and Lt Erv Copus, AFWL/DYV, for technical direction, and to Mr. Donald J. Kohler, Contracting Officer DNA/SPAS, for numerous helpful suggestions. In addition, the authors wish to thank Dr. E. A. Fitzgerald of McDonnell Douglas Astronautics Co. for his support and suggestions throughout this study effort, and Mr. Richard F. Walz of Lockheed Missiles and Space Company for his help in the reentry vehicle assessment effort.

For convenience, this final report has been organized in two volumes: Volume 1, Applications (Classified Secret); and Volume 2, Methods (Unclassified). The authors responsible for the individual sections of the report are as follows: Volume 1, Sections 1 and 2, L. H. Donahue, Section 3, G. M. Teraoka; Volume 2, Sections 1 to 3, L. H. Donahue, Section 4, J. J. Farrell. In addition, significant contributions were made to all sections by Dr. Farrell.

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1.0 INTRODUCTION

1.1 Background

The purpose of the study reported here was to develop and explore new techniques and procedures to aid in the design and assessment of reentry vehicles for survival against X-rays from exoatmospheric nuclear bursts. At present, a primary source of design information for reentry vehicle hardness against X-rays is underground nuclear testing. With the advent of a comprehensive nuclear test ban treaty, this source of proof data will no longer be available.

To prepare for the day when underground nuclear test data would no longer be available, Defense Nuclear Agency (DNA) initiated a program to develop a Nuclear Hardness Evaluation Procedure (NHEP). From this program has evolved the NHEP methodology, developed by Effects Technology, Inc., and a consortium of cooperating Agencies and Contractors. This methodology consists of a step-by-step general procedure for assessing reentry vehicle hardness, supported by a nuclear weapons effects data base.

An important tool which has been utilized to demonstrate the effectiveness of the NHEP methodology is the TRW-developed Failure Analysis by Statistical Techniques (FAST) code. The final step in the NHEP methodology is the integration of the data generated and compiled using this procedure, and the evaluation of reentry vehicle hardness at a prescribed confidence level. The FAST code is the analytical tool selected for this task.

The FAST code calculates the system probability of survival and associated confidence level for a system subjected to a defined environment. Hardness probability equations specified for the components of a system are integrated by a series of Monte Carlo calculations to provide a hardness probability statement for the system. The FAST code provides the means by which these data on reentry vehicle component hardness and hardness uncertainty can be combined to estimate the vehicle hardness level and the associated probability of survival and confidence level.

During two previous studies, TRW developed the use of FAST as a final step in the NHEP methodology. A major contribution to those efforts was made by McDonnell Douglas Astronautics Company (MDAC), by applying the Nuclear Hardness Evaluation Procedures to determine the reentry vehicle component hardnesses, and the uncertainties in these hardness estimates.

These data were then used by TRW to develop probabilistic models of component hardnesses and to integrate these hardness models with the FAST code to obtain vehicle system hardness estimates with their associated probability of survival and confidence levels.

In addition, new design optimization techniques were developed to identify optimal vehicle design configurations to assure maximum system hardness or minimum system weight at a prescribed probability of survival and confidence level. These optimization techniques were incorporated into a new computer code, BASH (Balanced System Hardness), and the utility of these design optimization techniques was verified by subsequent review of the BASH optimized design configurations, both by FAST code assessment and by engineering analysis.

1.2 Scope and Objectives

The present study is an extension of those previously reported NHEP efforts. In the current study, the FAST methodology was extended to provide new techniques for estimating the potential benefits of testing under uncertainty.

A new computer code named ELF, Evaluation of Likelihood Function was developed during the study to provide a thorough evaluation of test results. To provide a demonstration of the ELF code, this code was used to assess the graphite resin magnetic flyer plate test data. In addition, as a part of this contractual effort, TRW provided consultation and support to Lockheed Missiles and Space Company (LMSC) in the FAST assessment of a Reference reentry vehicle design.

For convenience, the final report of these activities has been subdivided into two volumes. Volume I, Applications, describes the results of the evaluations performed on the graphite resin magnetic flyer plate tests and their impact on predictions of reentry system performance. Also included in Volume I are the results of the analyses performed on the LMSC Reference reentry vehicle. Volume I is classified Secret.

Volume II of the Final Report, Methods, describes in detail the development of the statistical and analytical techniques that were evaluated during this study, and the methods that were incorporated into the ELF code. Also reported in this volume is a new version of the FAST code, FAST7, Conditional Probability FAST, which was developed during this effort, and which shows great promise for test planning and evaluation.

2.0 TEST PLANNING AND EVALUATION

Test planning and evaluation is an important area of decision making under uncertainty. The reentry vehicle designer is often faced with the weighty question of whether to proceed with a particular test or test series or not. To answer this question, he must be able to determine both the cost of the test, and the cost impact of the probable improvement in system performance that can be realized as a result of the tests.

Fortunately, these two aspects of the problem are separable, and can be decoupled without seriously degrading the outcome of the analysis. The determination of testing costs is accomplished by cost estimators using standard estimating techniques. Likewise, the cost savings that result from system performance increases involve the judgment of trained cost estimators. The present study does not address the issue of cost estimating, but rather is intended to provide a methodology to assess in a meaningful way the improvement of system performance that may result as a consequence of testing.

In an uncertain world, the performance of a system may be specified in terms of its ability to survive under a given load, expressed as probability of survival and associated confidence level, and the system cost or weight required to achieve this survivability. An improvement in system performance may be expressed in terms of an improvement in any of these criteria.

The complete characterization of the probabilistic response of a typical system or component to a univari te stimulus is given by the response surface illustrated in Figure 1. This surface shows the probability of survival, P_s, of thesystem as a function of the load, S, or stimulus level applied to the system, over a range of confidence levels, C.

Confidence is the perceived probability that at least the proportion of the population indicated will survive the given load shown by the corresponding probability-versus-load plane. The best estimate curve of probability of survival versus load is usually taken to be at the 50-percent confidence plane. In Figure 1, probability-of-survival-versus-load planes are indicated for confidence values of 0.10, 0.50, and 0.90. The projection of these intersections onto a single plane produces the useful plot shown on the right side of Figure 1.

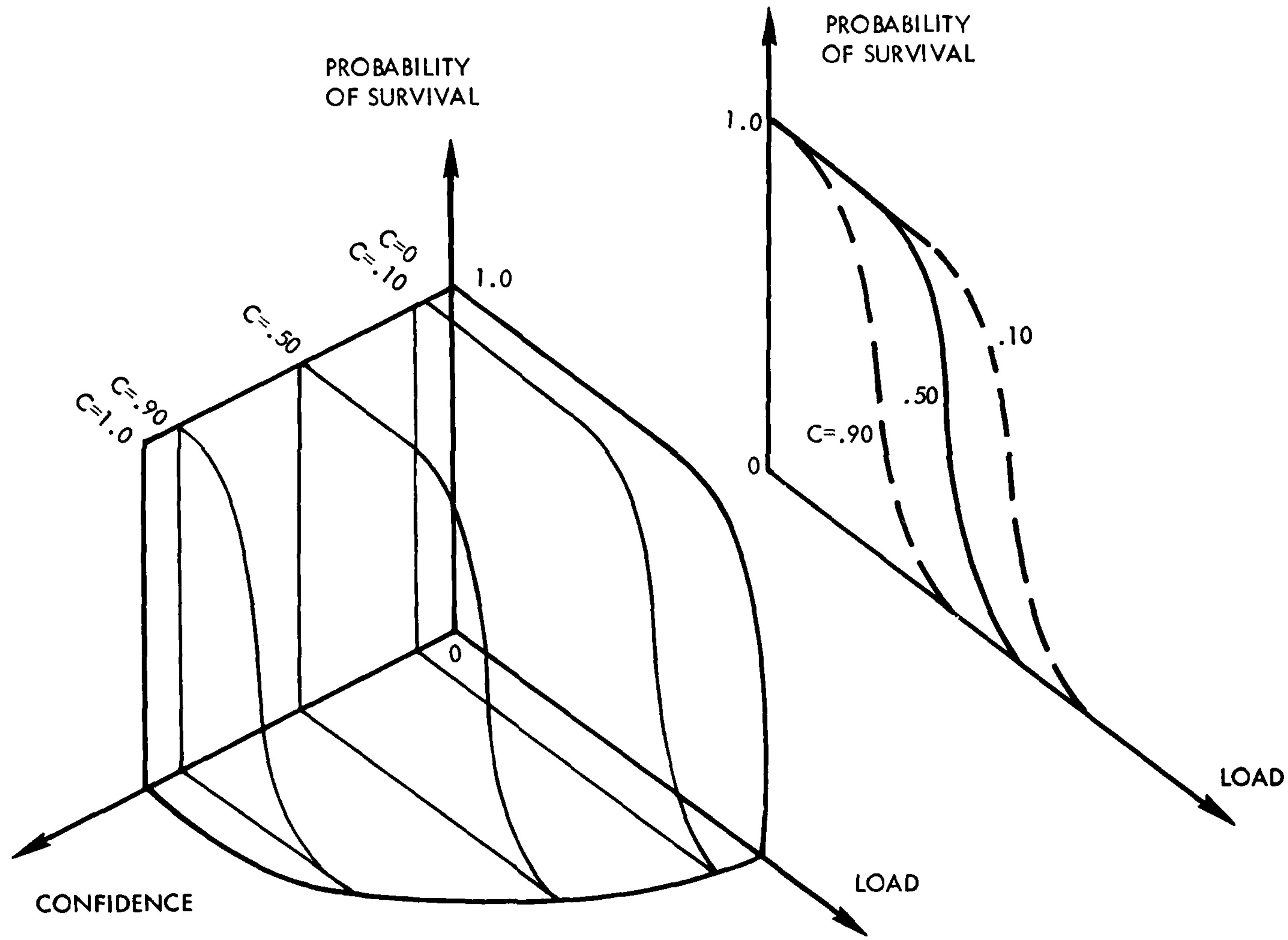


Figure 1. Probabilistic Response Surface

The response surface shown in Figure 1 is predicated on the premise that there are two distinct types of uncertainties affecting the response of the system: uncertainties which are random, and uncertainties which are systematic, and that random uncertainties determine the probability of survival of the system modeled, while systematic uncertainties affect the confidence ascribed to that probability of survival. Although this representation of the consequences of variation analysis is admittedly simplistic, experience has demonstrated that, in practice, it provides a useful method of accounting for the effects of uncertainties.

Random uncertainties are the result of the many indeterminable differences that occur from one system to the next, or from one run to the next. These differences exist at some level no matter how carefully the system is specified, or how rigorously quality control is excerised in the manufacture or operation of the system.

Systematic uncertainties in the response of the system exist because of a lack of complete knowledge of the underlying processes affecting the variable under analysis. If this ignorance could be reduced by additional, more refined research or testing, or by implementing more sophisticated analytical procedures, the amount of systematic uncertainty affecting the probability of survival could be reduced.

Random and systematic uncertainties have distinctly different effects on the survivability of the system. Random variations in system properties cause some systems to be harder, and others to be softer. These variations tend to average out when the probability of survival of a large number of systems is considered. Thus, while random variations cause a dispersion in the hardness distribution, they do not generally affect the central tendency of the distribution.

On the other hand, systematic uncertainties represent potential differences between the real performance of a system and its predicted performance based on analytical models and model parameter values. These potential differences dictate the probability of a bias or shift in the hardness distribution. Thus, because systematic uncertainties obscure the central location of the hardness distribution, these uncertainties reduce the system confidence shown in Figure 1.

The first step in the analysis of the benefits resulting from testing is the selection of possible tests to be considered. The possibilities include both system tests and component tests. However, it is nearly always true that component tests will be less expensive than system tests. Thus, the designer will ordinarily elect to analyze tests of components, searching especially for those components having large systematic uncertainties which intuitively would tend to have the greatest effect on system uncertainty.

When candidate tests have been selected for analysis, sensitivity studies are performed to estimate the impact on system performance of each of the tests selected. Using the FAST code, the effect of the component uncertainties on system performance is simulated by making a series of runs while varying the uncertainties of the selected component.[1]

These sensitivity studies will provide results which may be expressed, for example, as shown in Figure 2. For a constant level of probability of survival, confidence, and load, the cost or weight of the system will decrease as systematic, or bias, uncertainty for a given component decreases. Thus, by decreasing the systematic uncertainty associated with the component, it is possible for the system to display a lower cost at the same load level, or the same cost or weight at a higher load level.

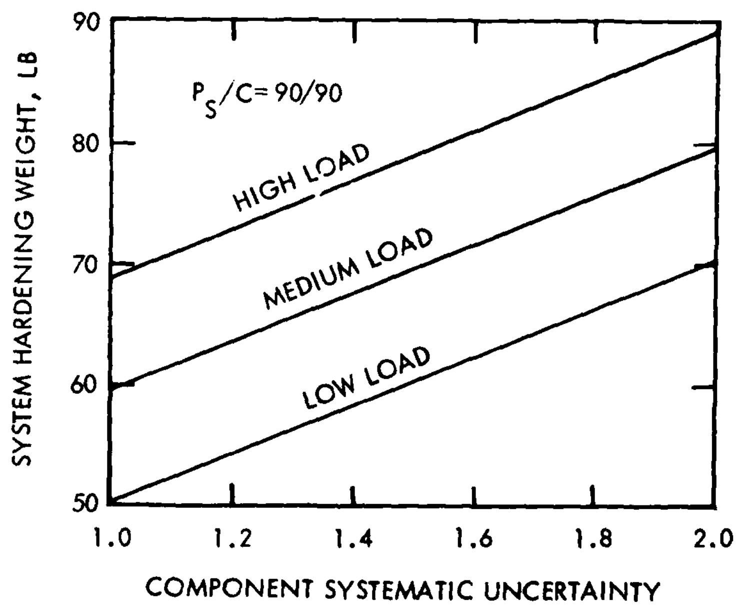


Figure 2. Component Uncertainty Sensitivities

An alternative is the use of the Balanced System Hardness (BASH) code for performing the sensitivity analyses without the need for the Monte Carlo procedure which is the basis of the FAST code. The BASH code, besides being more quick and efficient, provides directly an optimization of the mix of components in the final system design.[2]

To determine the utility of a given test, it is then necessary to estimate the change in component systematic uncertainty that would result from additional test data. By the use of the sensitivity plots of Figure 2, the system improvement can be evaluated as a function of these changes in component uncertainty.

Two different approaches for the analysis of the effects of testing were investigated during this study. The first is a method for analysis of test data that utilizes maximum likelihood estimation of the distributional parameters. This method is embodied in a new computer code named ELF, Evaluation of Likelihood Function. The second approach involves the use of conditional probability, and has been incorporated into a new version of the FAST code named FAST7.

Both of these computer codes can be used to objectively evaluate systematic uncertainties based on test results. The ELF code can be used to analyze both attributes (pass/fail) and variables test data so long as the test simulation is perfect. Where there is uncertainty in the simulation of the operation conditions by the test, this additional uncertainty must be accounted for outside the ELF code, as discussed below.

The FAST7 code is a powerful tool for analyzing the effects of pass/fail data, and it does not require perfect test simulation. In addition, FAST7 can be used at subsystem, or system levels, and can analyze correlations between component responses.

3.0 PARAMETRIC ANALYSIS METHOD

The first requirement for a testing cost/benefit analysis is to establish reliable, objective techniques for reduction of raw test data to usable statistical models. The criteria for measuring the success of the data reduction techniques include the following: (i) they must produce results which can be reproduced by workers throughout the community; (ii) they must be stable in the sense that they will yield reasonable results even if only limited data are available; and (iii) they must provide results in a form that are compatible with cost/benefit modeling.

One useful methodology developed during this study which meets the criteria stated above is the so-called parametric analysis described here. This method depends upon the estimation of the statistical parameters which determine the distribution of the test variables. A later section of this report will discuss a method which is not dependent on the estimation of parameters to determine distributional information regarding the test variables. This is the conditional probability analysis method.

In general terms, the parametric analysis method is based upon the construction and application of mathematical models of the results of repetitive experiments. These models make possible the study of the properties of the experiment, and predictions about the outcome of future trials of the experiment.

The conclusions drawn from this use of mathematical modeling are statistical inferences, and the process involved is decision-making based upon probability. Most of the decisions or inferences made by statisticians fall into one of two categories; either they involve the estimation of the parameters of a frequency function, or they involve testing some distributional assumption about the frequency function selected for the model. In the parametric analysis method developed during this study, both types of statistical decisions are utilized in analyzing test data.

The method that is usually preferred by most statisticians to construct a point estimate of a parameter of a frequency function of a random variable from several independent observations is the maximum likelihood method. By definition, the maximum likelihood estimator of a parameter of a frequency function is the estimator that maximizes the likelihood

function in terms of that parameter. The likelihood function is simply the joint frequency function of the several independent observations, formed by multiplying together the frequency functions of each observation.

A statistical test of a distributional assumption provides an objective technique for assessing whether an assumed distributional model provides an adequate discription of observed data. The following basic steps are usually involved: (1) a figure of merit, or modulus, known as a test statistic is calculated from the observed data; (2) the probability of obtaining the calculated test statistic, assuming the selected distributional model is correct, is determined; and (3) if the probability of obtaining the calculated test statistic is low, it is concluded that the assumed distributional model does not provide an adequate representation of the data.

The definition of low depends on the user's preferences and the consequences of rejecting the model. A probability of 0.10 or less is usually said to be low. If the probability associated with the test statistic is not low, then the data provide insufficient evidence that the assumed model is adequate.

Note that although this procedure permits a distributional model to be rejected as inadequate, it does not prove that the model is correct. The outcome of a statistical test is highly dependent upon the amount of available data: the more data there are, the better are the chances of identifying an inadequate model. If too few data points are available, even a distributional model that deviates grossly from the assumed model frequently cannot be established as inadequate.

Finally, after point estimates of the distributional parameters have been determined, and the distributional hypotheses utilized in the analysis have been tested, confidence bounds on the estimated distribution can be computed based on the test data sample size and behaviour. These confidence bounds act as a buffer on the estimated distribution function to account for uncertainties in the estimation of the parameters.

The parametric analysis methods outlined here were developed and codified in the Evaluation of Likelihood Function (ELF) computer code during this study. The development of the methodology, and its application to a typical data analysis problem are described in the following sections of this report.

3.1 MAXIMUM LIKELIHOOD ESTIMATION

Typically, experimental data may be categorized as one of two types, either quantitative or quantal. Quantitative data are those for which both the stimulus, or load, applied to the test specimen, and the response of the specimen, can be measured quantitatively; quantal, or attributes, data are those data for which the response is not quantitative, but rather is of the all-or-nothing, pass-or-fail, type.

Quantitative data are comparatively easy to analyze statistically because the stimulus for each data point is associated with a specific quantitative response. Quantal data, on the other hand, present special problems in analysis because the responses are not proportionally related to the stimuli. In addition, quantal data are less powerful in the sense that each response contains less information than a corresponding quantitative test point.

In quantal analysis, each individual test specimen has a particular stimulus level, called a tolerance, at which it will fail. If a particular specimen is tested at a stress level below its tolerance level, it will survive and if it is tested at a stress level greater than its tolerance level, it will fail. The test data represent a sample from a total population of material specimens which have a continuously distributed tolerance level. It is this tolerance level distribution that determines the probability of failure, P, of a particular specimen at a given test level.

Tables 1 and 2 contain a summary of magnetic flyer impact test results for Tape Wound Carbon Phenolic (TWCP)/silicone rubber/0.120 graphite resin specimens. These data were assembled by E. A. Fitzgerald of McDonnell Douglas Astronautics Company from tests conducted as a part of the Graphite Resin Screening Program, the Dynamic/Degraded Properties Program, and the NHEP Program.[3] The test levels are given in terms of the front face pressure on the TWCP, which is an approximate measure of the tensile stresses inside the graphite resin composite. The criteria for pass and fail used by Fitzgerald in rating the performance of the specimens are: Pass, no more than one hairline crack or delamination visible in the substructure by optical or X-ray examination; and Fail, any damage more extensive than that described above, including multiple hairline cracks or delaminations, single open cracks, or detached spalls.

Table 1. Graphite Resin Test Data

TEST	PRESSURE S	LOG(S)	RESULT	DESCRIPTION
1	2.4000	.87547	PASS	FLAT, GDA EPOXY (GRSP-2)
2	3.7000	1.30833	PASS	FLAT, GDA EPOXY (GRSP-2)
3	5.2000	1.64866	FAIL	FLAT, GDA EPOXY (GRSP-2)
4	2.4000	.87547	PASS	FLAT, GDA EPOXY (GRSP-2)
5	3.7000	1.30833	PASS	FLAT, GDA EPOXY (GRSP-2)
6	5.2000	1.64866	PASS	FLAT, GDA EPOXY (GRSP-2)
7	2.4000	.87547	PASS	FLAT, MDAC EPOXY (GRSP-2)
8	3.7000	1.30833	PASS	FLAT, MDAC EPOXY (GRSP-2)
9	5.2000	1.64866	PASS	FLAT, MDAC EPOXY (GRSP-2)
10	4.9000	1.58924	FAIL	FLAT, GDA EPOXY (GRSP-3)
11	4.9000	1.58924	FAIL	FLAT, GDA EPOXY (GRSP-3)
12	3.7000	1.30833	FAIL	FLAT, GDA EPOXY (GRSP-3)
13	3.7000	1.30833	PASS	FLAT, GDA EPOXY (GRSP-3)
14	4.9000	1.58924	PASS	FLAT, MDAC EPOXY (GRSP-3)
15	6.0000	1.79176	PASS	FLAT, MDAC EPOXY (GRSP-3)
16	4.9000	1.58924	PASS	FLAT, MDAC EPOXY (GRSP-3)
17	8.7000	2.16332	FAIL	FLAT, MDAC EPOXY (GRSP-3)
18	2.4000	.87547	PASS	FLAT, MDAC POLYIMIDE (GRSP-3)
19	2.9000	1.06471	PASS	FLAT, MDAC POLYIMIDE (GRSP-3)
20	4.9000	1.58924	PASS	FLAT, MDAC POLYIMIDE (GRSP-3)
21	8.7000	2.16332	FAIL	FLAT, MDAC POLYIMIDE (GRSP-3)
22	6.0000	1.79176	FAIL	FLAT, MDAC POLYIMIDE (GRSP-3)
23	2.4000	.87547	PASS	RING, GDA EPOXY (GRSP-3)
24	4.9000	1.58924	FAIL	RING, GDA EPOXY (GRSP-3)
25	3.7000	1.30833	PASS	RING, GDA EPOXY (GRSP-3)
26	2.4000	.87547	FAIL	RING, MDAC EPOXY (GRSP-3)
27	4.9000	1.58924	FAIL	RING, MDAC EPOXY (GRSP-3)
28	3.7000	1.30833	FAIL	RING, MDAC EPOXY (GRSP-3)

Table 1. Graphite Resin Test Data (Cont'd)

TEST	PRESSURE S	LOG(S)	RESULT	DESCRIPTION
29	3.7000	1.30833	FAIL	RING, MDAC POLYIMIDE (GRSP-3)
30	1.9000	.64185	PASS	FLAT, MDAC EPOXY (DDPP)
31	1.9000	.64185	PASS	FLAT, MDAC EPOXY (DDPP)
32	1.9000	.64185	PASS	FLAT, MDAC EPOXY (DDPP)
33	1.9000	.64185	PASS	FLAT, MDAC EPOXY (DDPP)
34	1.9000	.64185	PASS	FLAT, MDAC EPOXY (DDPP)
35	1.9000	.64185	PASS	FLAT, MDAC EPOXY (DDPP)
36	2.8000	1.02962	PASS	FLAT, MDAC EPOXY (DDPP)
37	2.8000	1.02962	PASS	FLAT, MDAC EPOXY (DDPP)
38	3.6000	1.28093	PASS	FLAT, MDAC EPOXY (DDPP)
39	2.9000	1.06471	PASS	FLAT, MDAC EPOXY (DDPP)
40	2.9000	1.06471	PASS	FLAT, MDAC EPOXY (DDPP)
41	2.4000	.87547	PASS	ARC, MDAC EPOXY (NHEP)
42	2.9000	1.06471	FAIL	ARC, MDAC EPOXY (NHEP)
43	3.7000	1.30833	PASS	ARC, MDAC EPOXY (NHEP)
44	3.7000	1.30833	PASS	ARC, MDAC EPOXY (NHEP)
45	4.9000	1.58924	FAIL	ARC, MDAC EPOXY (NHEP)
46	2.4000	.87547	PASS	ARC, MDAC POLYIMIDE (NHEP)
47	2.9000	1.06471	FAIL	ARC, MDAC POLYIMIDE (NHEP)
48	3.7000	1.30833	PASS	ARC, MDAC POLYIMIDE (NHEP)
49	3.7000	1.30833	FAIL	ARC, MDAC POLYIMIDE (NHEP)
50	4.9000	1.58924	FAIL	ARC, MDAC POLYIMIDE (NHEP)
51	1.6000	.47000	PASS	RING, MDAC EPOXY (NHEP)
52	1.6000	.47000	PASS	RING, MDAC EPOXY (NHEP)
53	2.9000	1.06471	FAIL	RING, MDAC EPOXY (NHEP)
54	2.9000	1.06471	PASS	RING, MDAC EPOXY (NHEP)
55	3.7000	1.30833	FAIL	RING, MDAC EPOXY (NHEP)
56	3.7000	1.30833	FAIL	RING, MDAC EPOXY (NHEP)
57	2.9000	1.06471	PASS	RING, MDAC POLYIMIDE (NHEP)
58	2.9000	1.06471	PASS	RING, MDAC POLYIMIDE (NHEP)

Table 2. Summary of Graphite Resin Test Data

MFGR	MATL	FLATS	RINGS	ARCS	
MDAC	POLYIMIDE	(GRSP-3) 5 —— 5	(GRSP-3) 1 (NHEP) 2 —— 3	(NHEP) 5 —— 5	13
	EPOXY	(GRSP-2) 3 (GRSP-3) 4 (DDPP) 11 —— 18	(GRSP-3) 3 (NHEP) 6 —— 9	(NHEP) 5 —— 5	32
GDA	EPOXY	(GRSP-2) 6 (GRSP-3) 4 —— 10	(GRSP-3) 3 —— 3	-- —— 0	13
		33	15	10	58

NOTES:

GRSP-2 : GRAPHITE RESIN SCREENING PROGRAM, PHASE 2
GRSP-3 : GRAPHITE RESIN SCREENING PROGRAM, PHASE 3
DDPP : DYNAMIC/DEGRADED PROPERTIES PROGRAM
NHEP : POST-GRSP TESTS

As indicated in Table 2, the data consist of a total of 58 tests, with 33 flat, 15 ring, and 10 arc specimens having been tested. Two contractors are represented, McDonnell Douglas Astronautics Company (MDAC) with a total of 45 specimens, and General Dynamics (GDA) with 13 specimens. Forty-five of the specimens were fabricated from epoxy resin reinforced with graphite fibers, and 13 were made from graphite-reinforced polyimide.

The information recorded for each test point consists of the pressure level (S, kbar) induced in the material specimen, and the quantal response of the specimen, i.e., whether it passed or failed the applied stress. Also shown in Table 1 are the logarithms of the test pressure levels, and a description of the material sample.

Table 3 shows the sample data from Table 1 after it has been classified. Because only eleven test levels were represented in the raw data, it was possible to classify these data at point values of the independent variable (rather than within arbitrary cell boundaries). At each test level, Table 3 shows the test level, s, the logarithm of the test level, the number of specimens tested, N, the number of specimens failed, R, and the fraction

Table 3. Classified Graphite Resin Test Data

CLASSIFIED INPUT DATA:

GROUP	S	LOG(S)	N	R	PF
1	1.6000	0.47000	2	0	0.00000
2	1.9000	0.64185	6	0	0.00000
3	2.4000	0.87547	8	1	0.12500
4	2.8000	1.02962	2	0	0.00000
5	2.9000	1.06471	9	3	0.33333
6	3.6000	1.28093	1	0	0.00000
7	3.7000	1.30833	14	6	0.42857
8	4.9000	1.58924	9	6	0.66667
9	5.2000	1.64866	3	1	0.33333
10	6.0000	1.79176	2	1	0.50000
11	8.7000	2.16332	2	2	1.00000

			58		

of the specimens that failed, PF. Classification of the quantal data, and calculation of the fraction failed, makes it possible to plot these data in terms of stimulus (stress) and response (failure probability), as shown in Figure 3.

The next step in the statistical analysis of these data is to fit a predictive model to the data points shown in Figure 3. This is done by: (1) assuming a frequency function for the data; (2) estimating the parameters of the frequency function by the maximum likelihood method; and (3) testing the assumptions of the frequency function selected.

Maximum likelihood estimating consists of determining the maximum of the likelihood function, where the likelihood function is the joint probability density function for all the independent observations. For the classified data of Table 2, a Bernoulli process is assumed to model the response of the specimens at each test level, because each specimen is assumed to respond independently, with two possible outcomes for each sample. Then the probable function at any test level may be predicted by the binomial frequency function

$$f = \binom{n}{r} P^{r} Q^{n-r} \tag{1}$$

where: f = Probability function for the test level

n = Number of data points at test level

r = Number of failures at test level

P = Probability of failure at test level

Q = Probability of survival at test level

= 1 - P

The likelihood function, which is the joint probability function for all the levels tested, is expressed as the product of the binomial probability functions for each of the k test levels, or

$$L = \prod_{i=1}^{k} \binom{n}{r}_i P_i^{r} Q_i^{n-r} \tag{2}$$

A lognormal distribution was assumed for the population from which the test data shown in Table 1 were drawn. This assumption is later tested, as discussed below. Utilizing the notation of Abramowitz[(4)] and Aitchison[(5)], the following definitions are provided

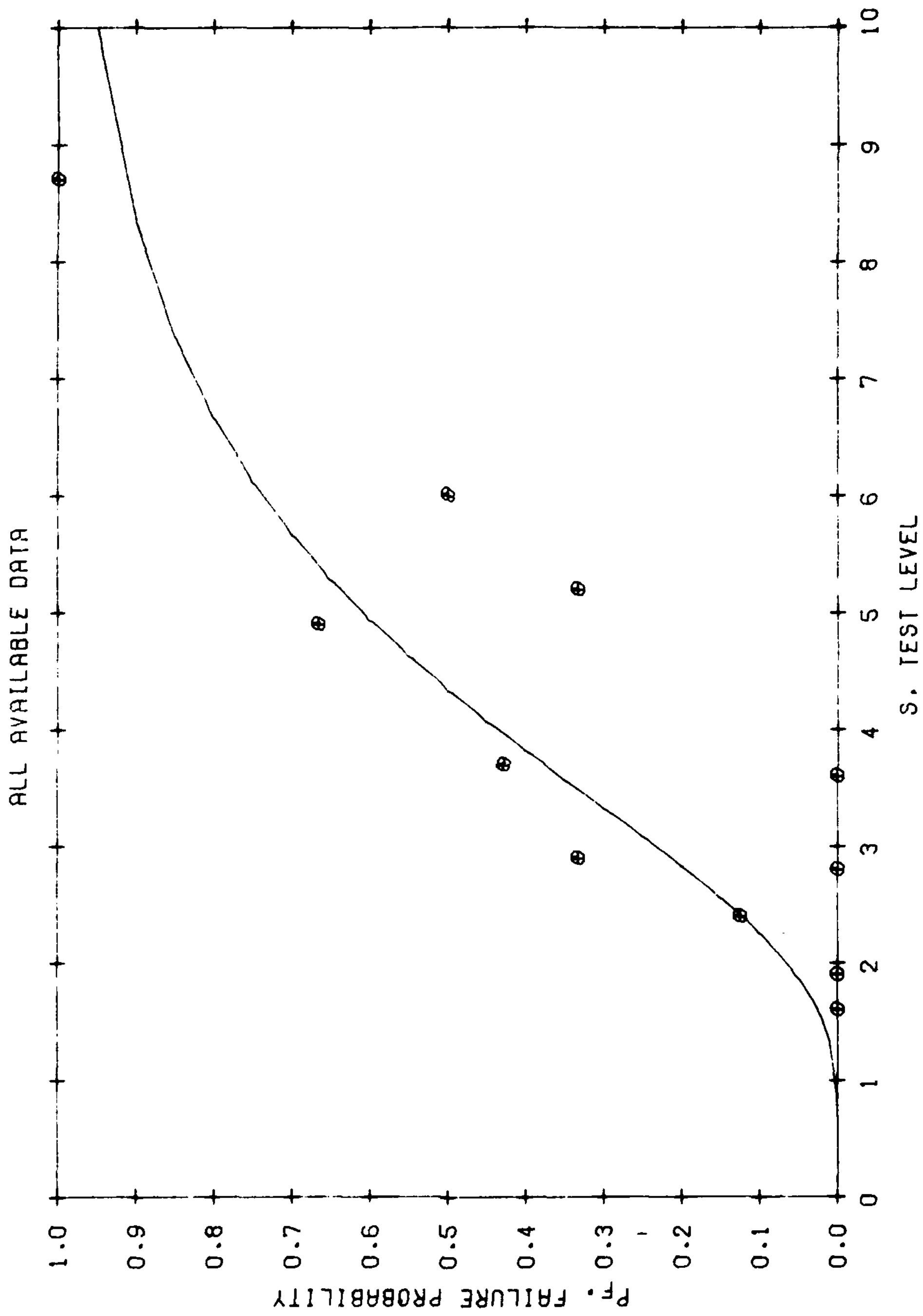


Figure 3. Analysis of All Available Data

$$x = \log_e s \tag{3}$$

where s = Test level

x = Logarithm of test level

For a lognormally distributed variable, s, the transform variable, x is normally distributed with mean, μ, and standard deviation, σ. It is convenient to introduce the standardized normal deviate

$$z = \left(\frac{x - \mu}{\sigma}\right) \tag{4}$$

which is normally distributed with a zero mean and a unit standard deviation.

The frequency function for the standardized normal deviate, z, is given by the familiar relationship

$$Z(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2} \tag{5}$$

and the probability of failure is given as the integral of equation 5, or

$$P(z) = \int_{-\infty}^{z} Z(t)\, dt \tag{6}$$

The likelihood function, L, given by equation 2 describes all the probable outcomes of sample testing in terms of the distributional parameters μ and σ. A typical likelihood function is illustrated in Figure 4. The maximum likelihood estimates of the distributional parameters are obtained by determining the maximum value of this function.

The classical method of determining the maximum likelihood estimates of the mean and standard deviation, $\hat{\mu}$ and $\hat{\sigma}$, consists of differentiating the likelihood function with respect to each of the parameters, setting the differentials equal to zero, and solving the resulting equations for the distributional parameters. That is, the equations

$$\frac{\partial L}{\partial \mu} = 0 \; ; \; \frac{\partial L}{\partial \sigma} = 0 \tag{7, 8}$$

are determined and solved for $\hat{\mu}$ and $\hat{\sigma}$.

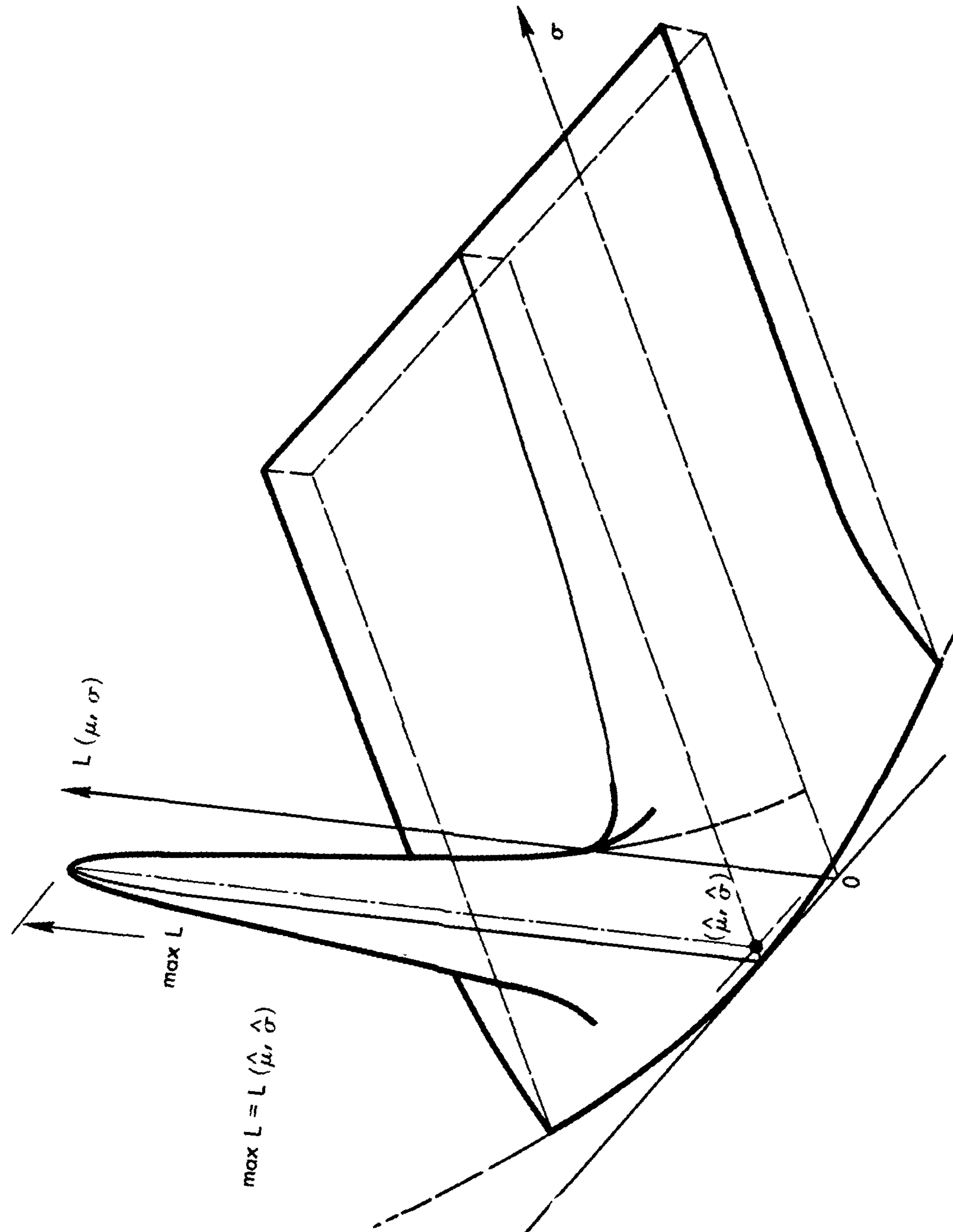


Figure 4. Typical Likelihood Function

Because equation 2 is not amenable to simple analytical methods of solution, it was found expeditious to solve these relationships by numerical methods. Two separate numerical methods were investigated during the study, the Probit Analysis method and a search technique. Computer routines were designed, implemented, and validated for each of these methods, and a combination of both methods was incorporated into the ELF code.

The name Probit Analysis has come to mean the method developed and popularized by Bliss[6] and Finney[7]. This method was originally confined to problems in biological assay, i.e., "the measurement of the potency of any stimulus, physical, chemical, or biological, physiological or psychological, by means of the reactions which it produces in the living matter"[7]. It is now recognized that the Probit Analysis method can be applied to any data for which appropriate data transformations can be determined[5, 8].

The basis of the Probit Analysis method is transformation of the test data in such a way that it can be fitted with a best (in the least squares sense) linear regression curve. More specifically, where quantal data are under analysis, the data are classified, and the fraction of samples failing in each class group are treated as the dependent variable.

Traditionally, the independent variable, or the logarithm of the variable when appropriate, has been taken to be normally distributed. Even though this assumption has long been associated with Probit Analysis, it is not clear that it is essential to the method. When the normal distribution is assumed, however, the values of the independent variable are conveniently transformed into standard normal deviates to linearize the data fit.

The method of solution utilized in the Probit Analysis method is facilitated by the introduction of two new parameters[5], which are defined as

$$\alpha = -\mu/\sigma \qquad (9)$$

$$\beta = 1/\sigma \qquad (10)$$

Using these new parameters, equation 4 is transformed to the linear relationship

$$z = \alpha + \beta \cdot x \qquad (11)$$

and Equations 7 and 8 are replaced by the relationships

$$\frac{\partial L}{\partial \alpha} = \sum_{i=1}^{k} \frac{nZ}{PQ} (r/n - P) = 0 \qquad (12)$$

$$\frac{\partial L}{\partial \beta} = \sum_{i=1}^{k} x \frac{nZ}{PQ} (r/n - P) = 0 \qquad (13)$$

which may be solved iteratively for the parameters α and β. Because of the popularity of this method, packaged computer routines for Probit Analysis are generally available in the scientific community[9].

A two-dimensional search technique, augmented by a quadratic predictor algorithm, was also developed which rapidly and efficiently finds the maximum likelihood estimates for μ and σ. The two methods of solution for the maximum likelihood estimates of the parameters of the tolerance distributions resulting from the test data were evaluated for accuracy and efficiency. It was found that the two techniques complement each other in the sense that where a particular data set gives one method convergence problems, the other method usually can converge quickly. For this reason, the ELF code contains a hybrid algorithm which utilizes both techniques.

3.2 HYPOTHESIS TESTING

Tests of the two distributional assumptions upon which the parametric analysis method is based are incorporated into the ELF code. The first distributional assumption is that at a given test level, each test may be considered to be the result of a Bernoulli process. That is to say, each test specimen responds independently, and with one of two possible outcomes, pass or fail, with probabilities P and Q. On this basis the test data at each test level may be described as being purely binomially distributed. This hypothesis is tested by the use of a chi-square test.

The second distributional assumption that was made was that the test samples were drawn from a population that is lognormally distributed. Although the assumption of this frequency function is not essential to the parametric analysis method, it is a convenient assumption and it will be

shown to have much utility for the data analyzed. The present version of the ELF code assumes the lognormal distribution, and tests this assumption by means of the Wilk-Shapiro W-Test.

3.2.1 Chi-Square Test

The most commonly used, and most versatile procedure for evaluating distributional assumptions is the chi-square goodness-of-fit test. To use this test, the data are grouped and compared to the expected number of observations based on the assumed distribution. The major advantage of the chi-square test is its versatility. It can be applied in a simple way to test any distributional assumption, without requiring knowledge of the values of the distribution parameters. Its major disadvantage is its lack of sensitivity in detecting inadequate models when few observations are available.

The chi-square statistic, X^2, is defined by the relationship

$$X^2 = \sum_{i=1}^{k} \frac{(o_i - e_i)^2}{e_i} \tag{14}$$

where k = number of pairs of frequencies to be compared

o_i = i th observed frequency

e_i = i th expected frequency

A value of X^2 of zero would correspond to exact agreement between observation and expectation, while increasingly large values of X^2 represent increasingly poor experimental agreement.

To provide a continuous measure of the goodness of fit implied by the X^2 statistic, it is necessary to introduce a new frequency function which approximates the frequency function of X^2 very well for large n. This new frequency function is obtained from the limit of the moment generating function of X^2 when n approaches infinity as

$$f(X^2 \mid \nu) = \left[2^{\frac{\nu}{2}} \cdot \Gamma(\tfrac{\nu}{2}) \right]^{-1} e^{-\frac{X^2}{2}} \left(X^2\right)^{\frac{\nu}{2} - 1} \tag{15}$$

The parameter ν is called the degrees of freedom because it represents the number of independent pairs being compared. Since the continuous chi-square distribution of equation 15 is only an approximation to the

discrete chi-square frequency function, judgment must be exercised with this test when very small samples are examined. Experience and theory indicate that the approximation is usually satisfactory, provided $e_i \geq 5$, and $k \geq 5$. Below these levels the chi-square test results are suspect[10].

The chi-square test possesses the useful property that it is applicable even when the cell probabilities depend upon unknown parameters, provided that the unknown parameters are replaced by their maximum likelihood estimates and one degree of freedom is deducted for each estimated parameter[10].

For the binomial relationship of equation 14, the chi-square statistic may be more easily calculated from the relationship

$$X^2 = \sum_{i=1}^{k} \frac{n(r/n - P)^2}{PQ} \tag{16}$$

which is chi-square distributed with k - 2 degrees of freedom[5].

The probability level associated with the distributional assumption is given by the integral of equation 15 as

$$Q(X^2 | \nu) = \int_{X^2}^{\infty} f(t | \nu)\, dt \tag{17}$$

This integral gives the probability of obtaining the calculated value of X^2, given ν degrees of freedom. Figure 5 shows a range of probability values given by equation 17 as a function of X^2 and ν. Equation 17 has been incorporated into the ELF code, providing an automatic evaluation of the chi-square test of a set of test data.

3.2.2 The Wilk-Shapiro W-Test

The W-test is an effective new statistical procedure for evaluating the assumption of normality (or lognormality) against a wide range of alternative distributions, even if only a relatively small number of observations are available. The test statistic is obtained by dividing the square of a combination of sample order statistics by the sample variance[11]. The W-statistic is given by the relationships

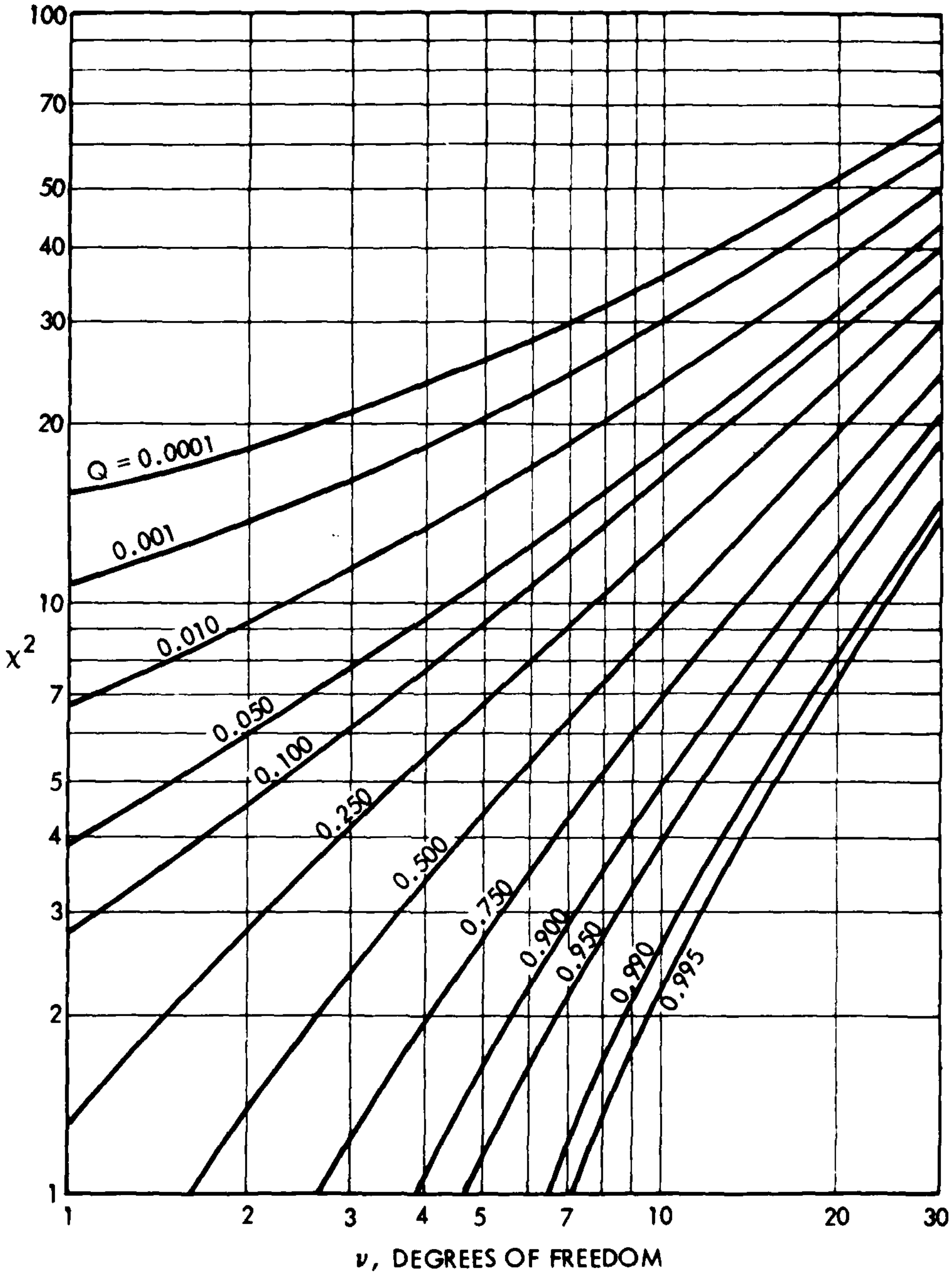


Figure 5. Chi-Square Probabilities

$$W = b^2/s^2 \tag{18}$$

$$b = \sum_{i=1}^{\ell} a_{n-i+1} (x_{n-i+1} - x_i) \tag{19}$$

$$s = \sum_{i=1}^{\ell} (x_i - \bar{x})^2 \tag{20}$$

where: x_i = ordered sample data such that $x_1 \leq x_2 \leq x_3 \leq \cdots \leq x_n$

ℓ = n/2 for n even

= (n-1)/2 for n odd

$\bar{x}$ = mean of data sample

and the values of the coefficients, a, are given in Table 4.

Figure 6 shows the relationship between sample size, W-statistic, and the probability of the calculated W-statistic. Note from Figure 6 that to obtain 0.50 probability, a W-statistic of greater than 0.90 is needed, even for the smallest sample size.

Hahn and Shapiro have approximated the results in Figure 6 with an empirical Johnson S_B distribution [12]. This approximation makes possible the calculation of the standard normal deviate, corresponding to the probability, P, of a given W-statistic by the equation

$$z = \gamma + \eta \log_e \left(\frac{W - \epsilon}{1 - W}\right) \tag{21}$$

where the constants γ, η, and ϵ are given in Table 5. This relationship allows the continuous calculation of the approximate probabilities associated with a given value of the W-statistic, and is incorporated into the ELF code.

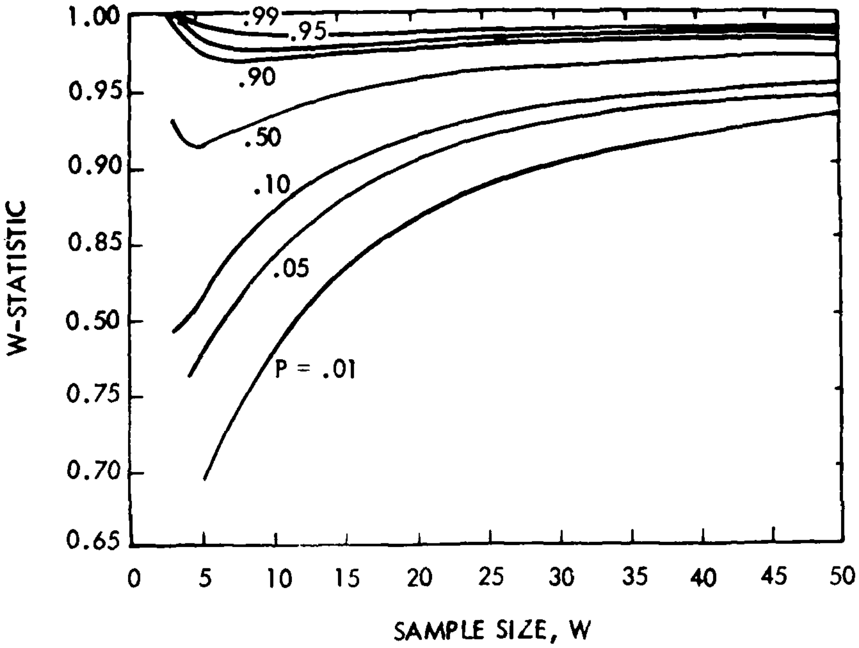
3.3 CONFIDENCE BOUNDS

In the parametric analysis method, confidence bounds on the estimated lognormal distribution are calculated from the test data by classical means. These confidence bounds account for the uncertainties in the estimates of the distributional parameters μ and σ that are the result of the sample size and behavior. These bounds assume that the test is perfectly calibrated, and that it simulates the operational conditions perfectly. Uncertainties in test calibration and simulation fidelity represent separate issues discussed later in this volume.

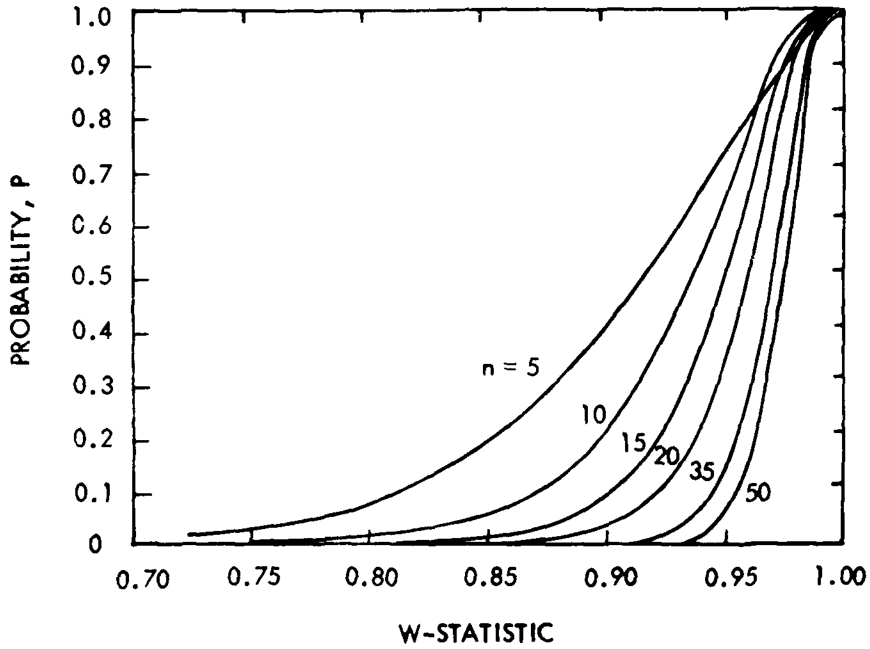
Table 4. Coefficients for Wilk-Shapiro W-Test

i \ n	3	4	5	6	7	8	9	10	11	12	13	14
1	0.7071	0.6872	0.6646	0.6431	0.6233	0.6052	0.5888	0.5739	0.5601	0.5474	0.5359	0.5251
2		0.1677	0.2413	0.2806	0.3031	0.3164	0.3244	0.3291	0.3315	0.3325	0.3325	0.3318
3				0.0875	0.1401	0.1743	0.1976	0.2142	0.2260	0.2347	0.2412	0.2460
4						0.0561	0.0947	0.1224	0.1429	0.1586	0.1707	0.1802
5								0.0399	0.0695	0.0922	0.1099	0.1240
6										0.0303	0.0529	0.0727
7												0.0240

i	15	16	17	18	19	20	21	22	23	24	25
1	0.5150	0.5056	0.4968	0.4886	0.4808	0.4734	0.4643	0.4590	0.4542	0.4493	0.4450
2	0.3306	0.3290	0.3273	0.3253	0.3232	0.3211	0.3185	0.3156	0.3126	0.3098	0.3069
3	0.2495	0.2521	0.2540	0.2553	0.2561	0.2565	0.2578	0.2571	0.2563	0.2554	0.2543
4	0.1878	0.1939	0.1988	0.2037	0.2059	0.2085	0.2119	0.2131	0.2139	0.2145	0.2148
5	0.1353	0.1477	0.1524	0.1587	0.1641	0.1686	0.1736	0.1764	0.1787	0.1807	0.1822
6	0.0880	0.1005	0.1109	0.1197	0.1272	0.1334	0.1399	0.1443	0.1480	0.1512	0.1539
7	0.0433	0.0593	0.0725	0.0837	0.0932	0.1013	0.1092	0.1150	0.1201	0.1245	0.1283
8		0.0196	0.0359	0.0496	0.0612	0.0711	0.0804	0.0878	0.0941	0.0997	0.1046
9				0.0163	0.0303	0.0422	0.0530	0.0618	0.0696	0.0764	0.0823
10						0.0140	0.0263	0.0368	0.0459	0.0539	0.0610
11								0.0122	0.0228	0.0321	0.0403
12										0.0107	0.0200



b. W-TEST STATISTIC



a. W-TEST PROBABILITIES

Figure 6. Wilk-Shapiro W-Test

Table 5. Empirical Constants for W-Test Probabilities

n	γ	η	ϵ	n	γ	η	ϵ
3	-0.625	0.386	0.7500	15	-4.373	1.695	0.2842
4	-1.107	0.714	0.6297	16	-4.567	1.724	0.2727
5	-1.530	0.935	0.5521	17	-4.713	1.739	0.2622
6	-2.010	1.138	0.4963	18	-4.885	1.770	0.2528
7	-2.356	1.245	0.4513	19	-5.018	1.786	0.2440
8	-2.696	1.333	0.4186	20	-5.153	1.802	0.2359
9	-2.968	1.400	0.3900	21	-5.291	1.818	0.2264
10	-3.262	1.471	0.3660	22	-5.413	1.835	0.2207
11	-3.485	1.515	0.3451	23	-5.508	1.848	0.2157
12	-3.731	1.571	0.3270	24	-5.605	1.862	0.2106
13	-3.936	1.613	0.3111	25	-5.704	1.876	0.2063
14	-4.155	1.655	0.2969				

For the case where the sampled population is normally distributed, confidence bounds on the estimation of μ and σ may be calculated by the use of the non-central t-distribution[13]. This distribution is based upon the non-central t parameter, t', which is defined by the equation

$$t' = \frac{z + \Delta}{\sqrt{X^2/\nu}} \tag{22}$$

where: z = standardized normal deviate (see equation 4)
Δ = non-centrality constant parameter
X^2 = chi-square parameter (see equation 14)
ν = degrees of freedom of chi-square parameter

The probability density function for non-central t is given by the expression

$$f(t'|\nu,\Delta) = \frac{\Gamma(\nu + 1)}{2^{\frac{1}{2}(\nu-1)}\,\Gamma(\frac{1}{2}\nu)\,\sqrt{(\pi\nu)}}\left(1+\frac{t'^2}{\nu}\right)^{-\frac{1}{2}(\nu+1)} \exp\left\{-\frac{1}{2}\,\frac{\nu\Delta^2}{(\nu+t'^2)}\right\} Hh_\nu\left\{\frac{-t'\Delta}{\sqrt{(\nu+t'^2)}}\right\}, \tag{23}$$

where $$Hh_\nu(x) = \int_0^\infty \frac{u^\nu}{\nu!}\exp\left\{-\tfrac{1}{2}(u+x)^2\right\} du \tag{24}$$

is the Hh function as defined by Fisher[13].

Given that the test data sample is drawn from a normal population with a mean, μ, and a standard deviation, σ, let $z(P)$ represent a standard normal deviate such that a proportion P of the population will lie below the limit $U(P) = \mu + z(P)\sigma$. If a random sample of n observations from the population is available, with a sample mean $\bar{x}$ and a standard deviation S, then a constant Kc can be defined such that 100C percent of the time $\mu + z(P)\sigma \leq \bar{x} + KcS$. In other words, there is confidence, C, that at least the proportion P of the population will lie below $\bar{x} + KcS$.

This probability/confidence statement may be expressed

$$C = P_r\left\{\mu + z(P)\sigma \leq \bar{x} + KcS\right\} \tag{25}$$

The quantity in braces may be rewritten as

$$\left[\frac{\frac{\mu - \bar{x}}{\sigma}\sqrt{n} + z(P)\sqrt{n}}{S/\sigma}\right] \leq Kc\sqrt{n} \tag{26}$$

so that the term to the left of the inequality is equivalent to non-central t defined in equation 22. Then equation 25 may be expressed as

$$C = P_r\left\{t' \mid \nu,\Delta \leq Kc\sqrt{n}\right\} \tag{27}$$

where

$$\Delta = z(P)\sqrt{n} \tag{28}$$

$$\nu = n - 1 \tag{29}$$

For convenience, Θ is used to define the operation

$$\Theta(t'_o \mid \nu,\Delta) = \int_{-\infty}^{t'_o} f(t'|\nu,\Delta)\, dt' \tag{30}$$

$$= P_r\left\{t' \leq t'_o\right\} \tag{31}$$

Then the inverse operation, Θ^{-1}, gives the value of t'_o corresponding to a specified probability level. The solution of equation 25 for Kc may now be written as

$$Kc = \frac{1}{\sqrt{n}}\Theta^{-1}(1 - C|\nu,\Delta) \tag{32}$$

This equation provides the standoff distance corresponding to a specified probability of failure (implied by Δ) and confidence level, C. Using the maximum likelihood estimates, $\hat{\mu}$ and $\hat{\sigma}$, as the sample parameters, the actual limit at any probability/confidence limit is given by

$$U(P,C) = \hat{\mu} + \frac{\hat{\sigma}}{\sqrt{n}}\Theta^{-1}(1 - C|\nu,\Delta) \tag{33}$$

It is interesting to note that the non-central t-distribution has not received wide use in the scientific community. This is partly due to the fact that this distribution is a fairly recent development. The definitive exposition of the theory was not presented until the work of Johnson and Welch was published in 1940[13]. The main reason this distribution has not been significantly utilized, however, is the difficulty encountered in evaluating the functions which define the non-central t-distribution.

The non-central t-distribution is often confused with the Student's t-distribution, which is a relatively simple function to evaluate. The Student's t-distribution is useful in evaluating the confidence limits of a distribution where the measure of dispersion, σ, is known, and only the measure of central tendency, μ, is subject to uncertainty. The Student's t-distribution is a special case of the non-central t-distribution, for the situation where Δ, the non-centrality constant, is equal to zero.

Modernly, the solution for the integral of the non-central t-distribution is available in many of the complete software libraries. However, no solutions for the inverse of the non-central t indicated in equation 31 were found during this study, either in software libraries or in the scientific literature. Solutions for both the probability integral and the inverse function were developed and incorporated in the ELF code during this study.

Numerical solutions for the probability integral of the non-central t-distribution and its inverse were developed by the following methods. First, the probability integral was evaluated by a method suggested by Owen[14]. The integral of equation 30 was integrated by parts repeatedly, yielding distinct expressions for even and odd degrees of freedom, ν. For even degrees of freedom, the terms resulting from the repeated integrations are evaluated by series expansion in terms of the standard normal integral. For odd degrees of freedom, the solution is expressed in terms of the standard normal integral function and the T-function defined by Owen[15]. An economical numerical solution for the Owen T-function was developed based on a Gaussian quadrative method given by Young and Minder[16].

An efficient numerical method was developed for the solution of the inverse of the integral of the non-central t-distribution involving secant-guided trial-and-error iterations. An initial estimate of the solution is provided by assuming that the sample is normally distributed, using a formula suggested by Johnson and Welch[13]. Subsequent estimates are calculated by the secant method, constrained by the false position, or regula falsi, technique. This trial-and-error method is greatly accelerated by transforming the values of the iterations into the normal probability plane. Even though this transformation is only approximate for the non-central t probability curves, it linearizes these S-shaped curves sufficiently so that the secant estimations converge quadratically.

Validation of the software resulting from these studies was accomplished by reproducing the tables of Resnikoff and Lieberman[17]. This volume is the standard resource for non-central t-distribution tables. Unfortunately, the accuracy of the integral tables presented is only four significant figures, and the tables of percentage points of t given were developed by inverse interpolation in the probability tables.

The ELF code rountines were also tested against tables by Owens[18], Bombara[19], Natrella[8], and Pearson[20]. Finally, the code was self-tested by calculating probabilities and then inverse probabilities, to see how close the routines could come to the starting point.

3.4 ELF CODE RESULTS

Figure 7 shows the ELF code output resulting from the analysis of the total quantity of data available, as given in Table 1. At the top of the page are listed the 11 test levels, or groups, into which the data were classified. Given in the listing are the group number, the test level, S, the logarithm of the test level, the total number of tests at this level, N, the number of failures at this level, R, and the fraction failed at this level, PF = R/N.

The maximum likelihood estimates of the parameters of the population represented by this sample are given at the center of the page. The median of the population, m, is estimated at 4.334 and the K-factor characterizing the random variations exhibited by the samples, K_R, is seen to be 2.719, the mean is given as 1.466, and the standard deviation is 0.510.

The median and the K-factor represent the measures of central tendency and spread of the data, respectively, in the natural, or untransformed, domain, and the mean and the standard deviation represent the same properties of the data sets in the logarithm domain. These parameters are defined mathematically by the following relationships

$$x = \log_e s \qquad (34)$$

where s = measure of load (here, pressure level, kbars)

x = logarithm of load

PROGRAM ELF
DATE: 30 MAR 79
TIME: 08.30.00.
ITEM: ALL AVAILABLE DATA

CLASSIFIED INPUT DATA:

GROUP	S	LOG(S)	N	R	PF
1	1.6000	0.47000	2	0	0.00000
2	1.9000	0.64185	6	0	0.00000
3	2.4000	0.87547	8	1	0.12500
4	2.8000	1.02962	2	0	0.00000
5	2.9000	1.06471	9	3	0.33333
6	3.6000	1.28093	1	0	0.00000
7	3.7000	1.30833	14	6	0.42857
8	4.9000	1.58924	9	6	0.66667
9	5.2000	1.64866	3	1	0.33333
10	6.0000	1.79176	2	1	0.50000
11	8.7000	2.16332	2	2	1.00000

			58		

MAXIMUM LIKELIHOOD PARAMETERS

MEDIAN = 4.33355 MEAN = 1.46639
K-FACTOR = 2.71935 STD DEV = 0.51041

GOODNESS-OF-FIT TESTS

CHI-SQUARE TEST OF BINOMIALITY OF DATA

CHI-SQUARE = 4.50737
DEG OF FREEDOM = 9

PROBABILITY = 0.87497

WILK-SHAPIRO TEST OF LOGNORMALITY OF DATA

W-STATISTIC = 0.98380
SAMPLE SIZE = 11

PROBABILITY = 0.98133

Figure 7. ELF Code Analysis Summary

For s lognormally distributed, x is normally distributed with a probability density function

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \tag{35}$$

The mean of the log of load, μ, is defined as

$$\mu = \int_{-\infty}^{\infty} x\ f(x)\ dx \tag{36}$$

and the standard deviation of the load, σ, is the square root of the variance

$$\sigma^2 = \int_{-\infty}^{\infty} (x - \mu)^2\ f(x)\ dx \tag{37}$$

The median, m, is given implicitly by the expression

$$0.5 = \int_{-\infty}^{m} f(x)\ dx \tag{38}$$

Finally, the K-factor, defined by the expression

$$K = \exp\ (1.96\sigma) \tag{39}$$

is a factor applied to the median to give the upper and lower bounds corresponding to 95 percent of the population.

The results of the tests of the distributional assumptions are shown at the bottom of the the page. The chi-square statistic used to test the assumption of binomiality of the performance of the specimens at each test level is seen to be equal to a value of 4.507. The probability of achieving a value of chi-square equal-to-or-greater than this value, with 9 degrees of freedom, is 87.5 percent. In other words, the odds against the binomiality assumption being correct for this data set are only 12.5 percent based on this test. The W-statistic computed for the data set is 0.984, and the probability level associated with this statistic, for a sample size of 11, is 98.13 percent. Thus, the tests of the distributional assumptions give little indication of nonconformity of this data set.

Figure 3 gives tabulated values of the estimated distribution of the probability of failure for the graphite resin test specimens. These values were computed from the sample by the ELF code for three confidence levels: 10, 50, and 90 percent. The 50-percent confidence values represent the best estimate lognormal distribution for this population. The 10 and 90-percent confidence values represent the systematic uncertainty based on the sample size and variation, assuming perfect test calibration and test simulation. These confidence bounds are theoretically exact, being computed by the use of the non-central t-distribution, as described above.

Figure 9 shows a fragility curve, i.e., a plot of probability of failure versus test level, for the test data of Table 1. This plot illustrates the grouped data points, listed in Table 3, together with the 10, 50, and 90-percent confidence curves tabulated in Figure 8. Also shown on this plot are the number of tests in each group, N, and the number of specimens which failed in each group, R.

For example, at the test level of 6 kbar, two specimens were tested, with one passing and one failing, giving an experimental failure probability of 50 percent. The best estimate curve, on the other hand, predicts a failure probability for the total population at this test level of 73.7 percent, or about three out of four. At the test level of 3.7 kbar, it is seen that 14 specimens were tested, of which 6 failed, giving an experimental failure rate of 42.9 percent. In comparison, the best estimate prediction of failure rate at this test level is seen to be 37.8 percent.

As intuition would predict, the larger the sample at any given test level, the more nearly the measured probability of failure matches the predicted probability of failure. With the maximum likelihood estimation method employed by the ELF code to analyze these test data, the best estimate distribution is optimized in the sense of minimizing the distance from each data point, on the basis of the number of tests at that point. In other words, the points seen plotted in Figure 3, each of which represents a group of test data, were weighted in the analysis according to the number of tests in the group.

MEDIAN = 4.3335 MEAN = 1.4664
K-FACTOR = 2.7194 STD DEV = 0.5104
SIZE = 58 GROUPS = 11

ALL AVAILABLE DATA

TEST LEVEL, S (UNCERTAINTY FACTOR, KB)

PROB (PF)	CONFIDENCE 10.0		50.0		90.0
0.10	0.7020	(1.2747)	0.8949	(1.1997)	1.0737
1.00	1.0899	(1.2125)	1.3215	(1.1584)	1.5308
2.00	1.2743	(1.1919)	1.5188	(1.1448)	1.7388
3.00	1.4067	(1.1794)	1.6590	(1.1366)	1.8857
5.00	1.6090	(1.1630)	1.8714	(1.1261)	2.1073
10.00	1.9763	(1.1399)	2.2528	(1.1116)	2.5043
15.00	2.2675	(1.1260)	2.5533	(1.1034)	2.8172
20.00	2.5269	(1.1162)	2.8205	(1.0979)	3.0966
25.00	2.7705	(1.1088)	3.0718	(1.0942)	3.3612
30.00	3.0068	(1.1030)	3.3166	(1.0917)	3.6208
35.00	3.2413	(1.0985)	3.5607	(1.0903)	3.8820
40.00	3.4780	(1.0951)	3.8087	(1.0896)	4.1502
45.00	3.7207	(1.0925)	4.0649	(1.0898)	4.4301
50.00	3.9729	(1.0908)	4.3335	(1.0908)	4.7270
55.00	4.2391	(1.0898)	4.6199	(1.0925)	5.0474
60.00	4.5251	(1.0896)	4.9307	(1.0951)	5.3995
65.00	4.8376	(1.0903)	5.2742	(1.0985)	5.7938
70.00	5.1866	(1.0917)	5.6624	(1.1030)	6.2457
75.00	5.5871	(1.0942)	6.1135	(1.1088)	6.7785
80.00	6.0645	(1.0979)	6.6583	(1.1162)	7.4320
85.00	6.6661	(1.1034)	7.3550	(1.1260)	8.2819
90.00	7.4991	(1.1116)	8.3360	(1.1399)	9.5024
95.00	8.9116	(1.1261)	10.0353	(1.1630)	11.6715
97.00	9.9591	(1.1366)	11.3200	(1.1794)	13.3505
98.00	10.8007	(1.1448)	12.3648	(1.1919)	14.7376
99.00	12.2677	(1.1584)	14.2106	(1.2125)	17.2301
99.90	17.4914	(1.1997)	20.9850	(1.2747)	26.7500

Figure 8. ELF Code Estimated Distribution

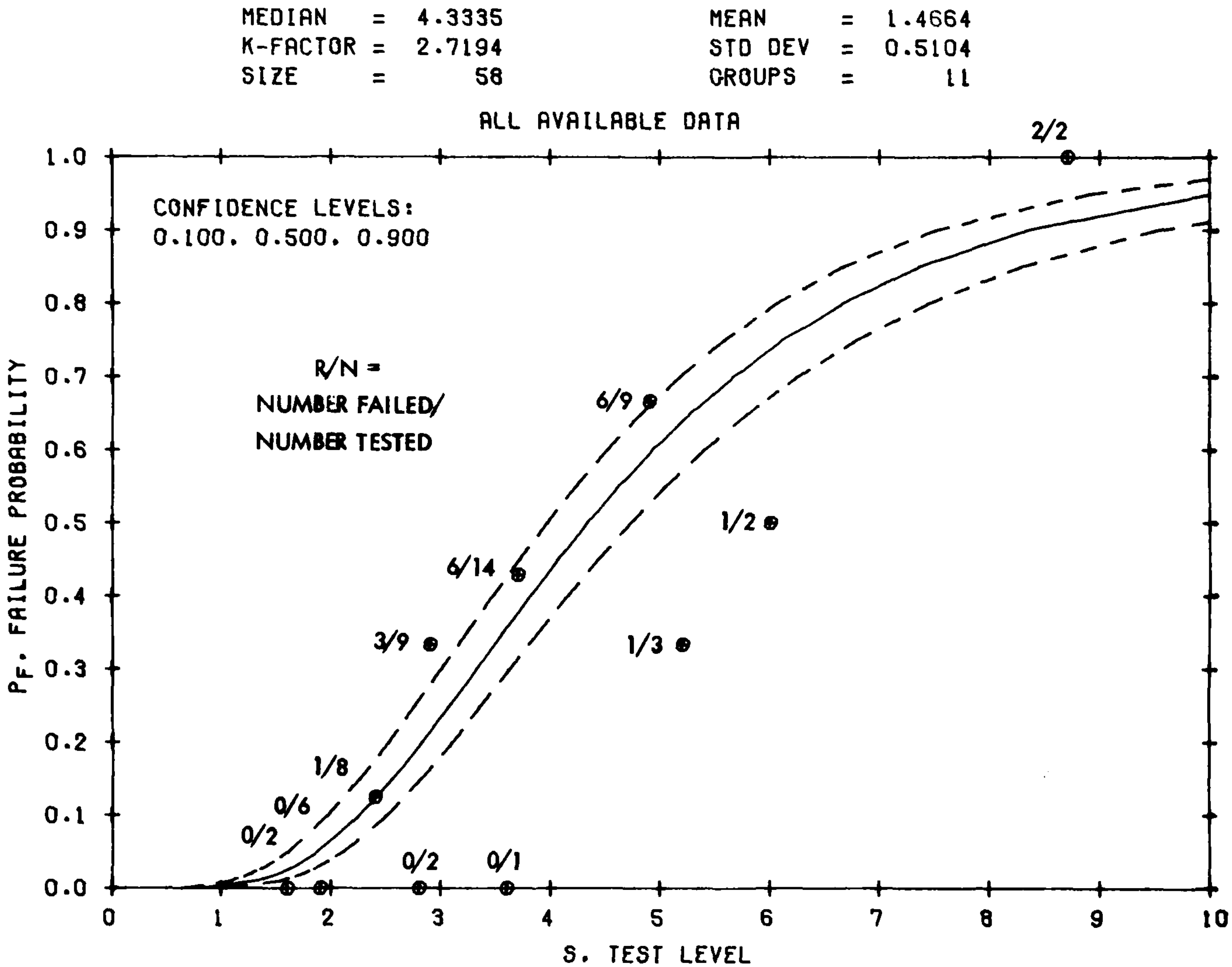


Figure 9. Fragility Curve for All Available Data

3.5 APPLICATION OF ELF RESULTS

The fragility of the graphite resin specimens given by the curves of Figure 9 represents a nearly complete description of the performance of this material based on the available test data. Random uncertainties in the material performance are specified by the random K-factor, K_R, listed at the top of the figure.

The confidence bounds shown in Figure 9 reflect a major portion of the systematic uncertainty affecting the performance of the material, i.e., the uncertainty resulting from the approximation due to sampling. However, there are additional systematic uncertainties that need to be accounted for in a complete fragility statement. These are the uncertainties due to test simulation fidelity and test calibration.

When the systematic uncertainties can be assumed to be lognormally distributed with constant parameters, these components of systematic uncertainty can be simply root-sum-squared together to provide the total uncertainty. A thorough discussion of the rationale behind this assertion is given in the report of the previous study(2).

The difficulty which arises in the implementation of this strategem is that the systematic uncertainty given by the non-central t-distribution is not constant over the range of probabilities, as shown in Figure 10. The factor which relates the confidence bounds varies continuously as probability of failure varies.

Figures 10 illustrates the variation in the confidence bounds given by the non-central t-distribution. Figure 10 is a listing of the 2.5, 50.0, and 97.5 percent confidence curves for the tests of MDAC graphite epoxy rings. In Figure 11, the data have been transformed to the logarithm/probability plane so that a lognormal curve appears as a straight line. The straight solid line in the center of the plot is the best estimate (50-percent confidence) lognormal distribution based on the maximum likelihood estimators printed at the top of the figure.

The curved solid lines in Figure 11 are the non-central t confidence bounds for 2.5- and 97.5-percent confidence (95 percent of the population included). Finally, the dashed lines in the figure represent constant K_B factor loci used to approximate the 2.5/97.5 confidence bounds. Note that the constant K_B factor which determines the position of the dashed lines is based on matching confidence bounds at the 2.5-percent confidence level and a probability of failure of 10 percent.

MEDIAN = 2.4378
K-FACTOR = 1.5786
SIZE = 9

MEAN = 0.8911
STD DEV = 0.2329
GROUPS = 5

MDAC EPOXY RINGS

TEST LEVEL, S (UNCERTAINTY FACTOR, KB)

PROB (PF)	CONFIDENCE 2.5		50.0		97.5
0.10	0.5915	(2.0062)	1.1868	(1.3008)	1.5437
1.00	0.8229	(1.7229)	1.4178	(1.2391)	1.7569
2.00	0.9244	(1.6343)	1.5108	(1.2196)	1.8426
3.00	0.9946	(1.5815)	1.5729	(1.2082)	1.9003
5.00	1.0980	(1.5135)	1.6618	(1.1939)	1.9841
10.00	1.2749	(1.4186)	1.8086	(1.1761)	2.1271
15.00	1.4064	(1.3616)	1.9149	(1.1680)	2.2366
20.00	1.5173	(1.3207)	2.0039	(1.1645)	2.3336
25.00	1.6165	(1.2889)	2.0835	(1.1642)	2.4256
30.00	1.7082	(1.2631)	2.1577	(1.1664)	2.5167
35.00	1.7948	(1.2418)	2.2287	(1.1708)	2.6093
40.00	1.8779	(1.2238)	2.2983	(1.1772)	2.7055
45.00	1.9588	(1.2087)	2.3676	(1.1856)	2.8070
50.00	2.0382	(1.1961)	2.4378	(1.1961)	2.9158
55.00	2.1171	(1.1856)	2.5100	(1.2087)	3.0339
60.00	2.1965	(1.1772)	2.5857	(1.2238)	3.1645
65.00	2.2775	(1.1708)	2.6664	(1.2418)	3.3111
70.00	2.3613	(1.1664)	2.7542	(1.2631)	3.4790
75.00	2.4500	(1.1642)	2.8523	(1.2889)	3.6763
80.00	2.5466	(1.1645)	2.9656	(1.3207)	3.9166
85.00	2.6571	(1.1680)	3.1034	(1.3616)	4.2256
90.00	2.7938	(1.1761)	3.2859	(1.4186)	4.6613
95.00	2.9953	(1.1939)	3.5762	(1.5135)	5.4125
97.00	3.1272	(1.2082)	3.7783	(1.5815)	5.9751
98.00	3.2252	(1.2196)	3.9336	(1.6343)	6.4288
99.00	3.3826	(1.2391)	4.1915	(1.7229)	7.2216
99.90	3.8496	(1.3008)	5.0075	(2.0062)	10.0461

Figure 10. Estimated Distribution for MDAC Epoxy Rings

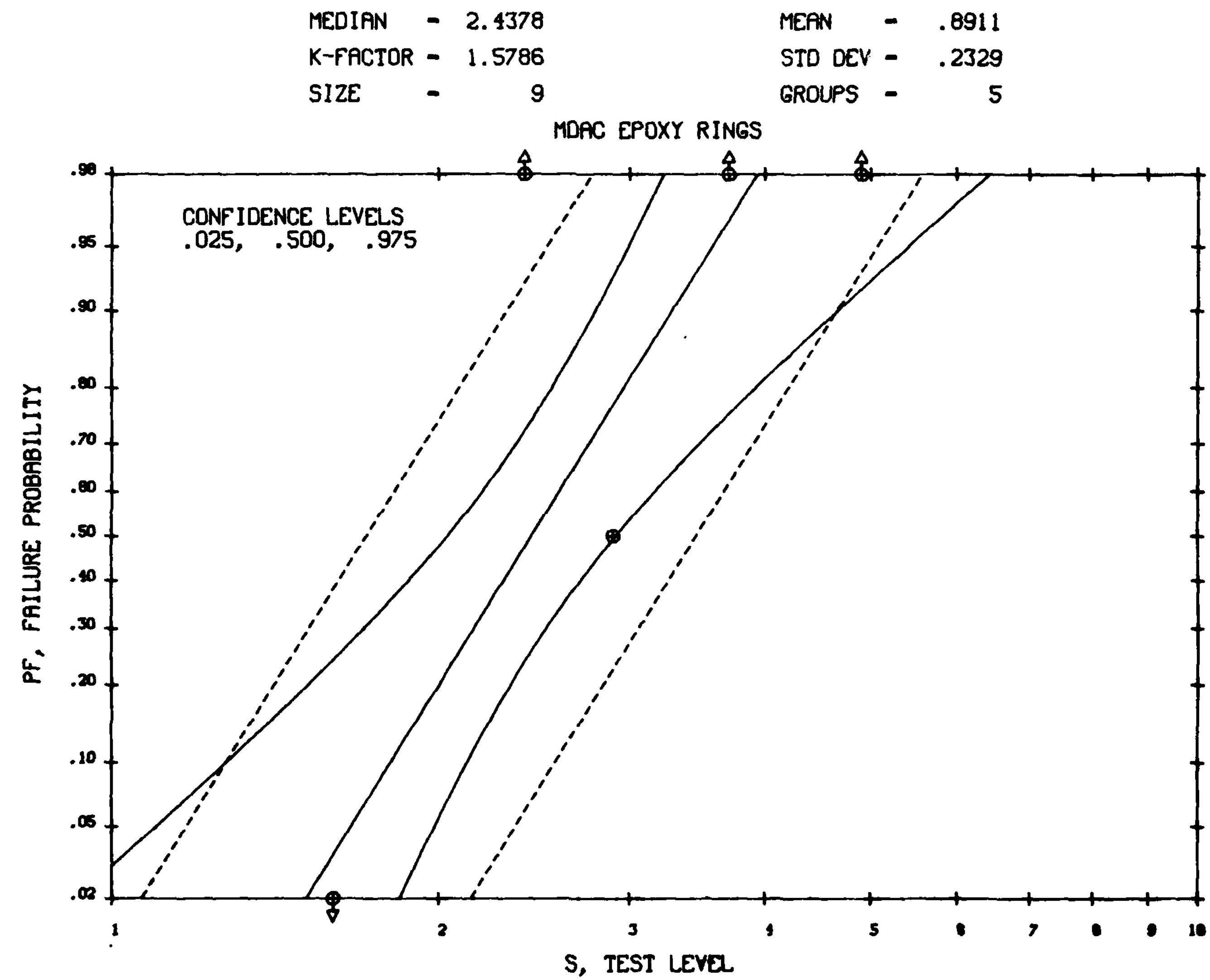


Figure 11. Transformed Plot of Constant K_B Matched at 10% P_F

The system designer will try to assure that the system will exhibit an adequate probability of survival at a medium to high level of confidence. Because Figure 11 is plotted in terms of failure probability, it is necessary to work on this plot with the complements of the desired values of probability and confidence. In other words, a desired probability-of-survival/confidence of 90/90 will be read on this plot as probability-of-failure/confidence of 10/10.

Confidence bounds of 2.5 and 97.5 percent are plotted in Figure 11 because these levels have been conventionally accepted as the levels used to define a constant K_B (see equation 39). When designing for confidence, it is likely the designer would work to less stringent confidence levels if possible.

Matching the constant K_B lines at 10-percent P_F causes an over-conservative bound at failure probability levels of 50 percent and above. On the other hand, at low probabilities of failure, the constant K_B bound is seen to be overly optimistic. The log-probability transform of Figure 11 is ideal for illustrating the difficulties that arise in trying to select a constant K_B. However, the presentation of these data in the natural, untransformed plane as shown in Figure 12 provides a better perspective of the tradeoffs involved. The discrepancies are here not so exaggerated as in the transform plane.

Figures 13 and 14 provide a similar comparison for constant K_B lines matched at a probability of failure of 20 percent, and 15 and 16 illustrate a match at 30 percent probability of failure. If it is remembered that the system designer is most concerned with a range of probabilities of failure from about 50 percent to 5 percent, the constant K_B line matched a probability of failure of 20 percent appears to provide an approximation that would be adequate for many design problems.

From Figure 10, the 95-percent included confidence factor at a probability of failure of 20 percent is seen to be 1.32. This is the approximate measure of systematic uncertainties due to sampling effects. Fitzgerald of MDAC estimated that differences between the pulse simulated by the magnetic flyer plate in the test, and the pulse induced by exoatmospheric X-rays would cause a systematic uncertainty factor of 1.20. He also estimated calibration uncertainties to yield a factor of 1.10.[2]

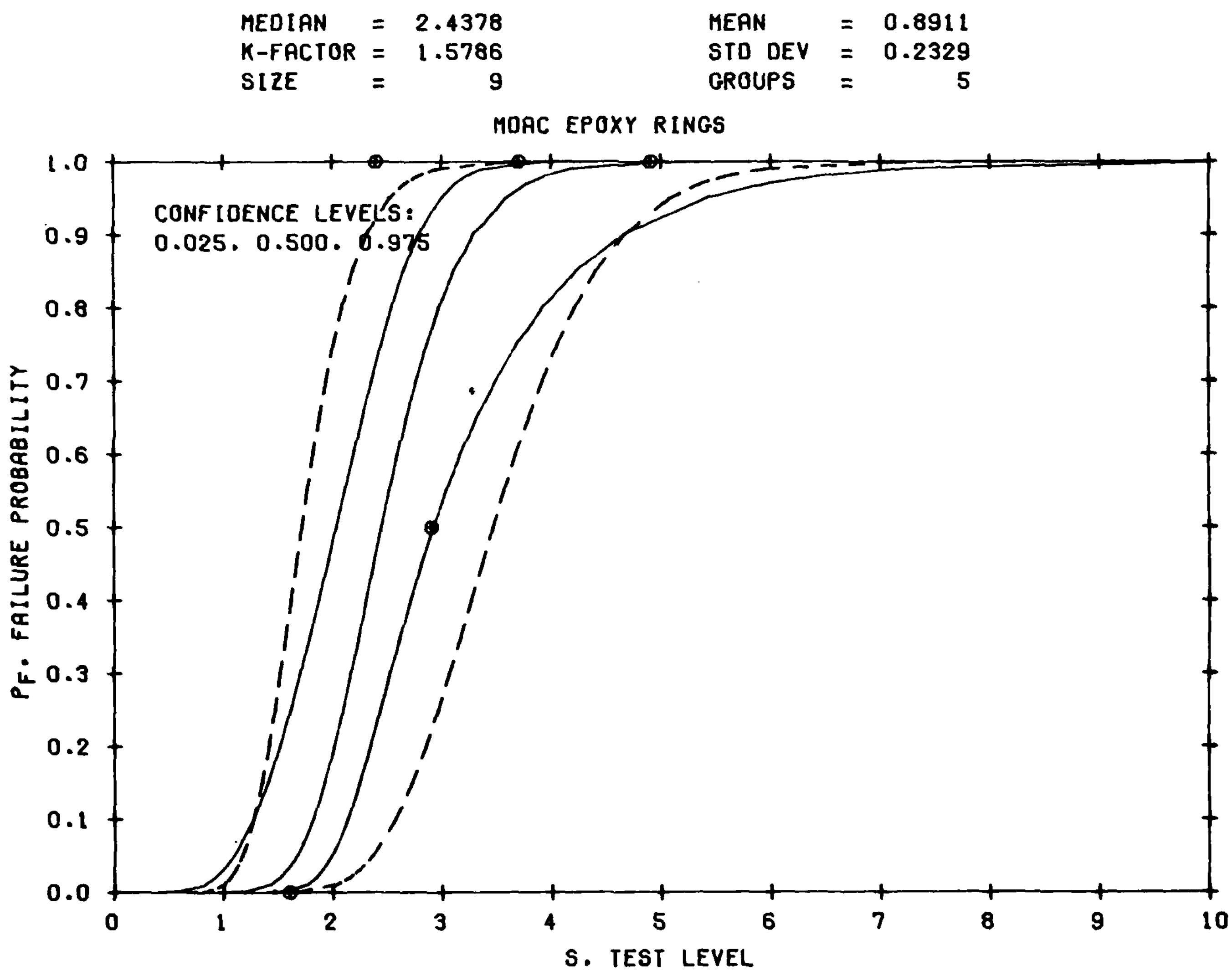


Figure 12. Natural Plot of Constant K_B Matched at 10% P_F

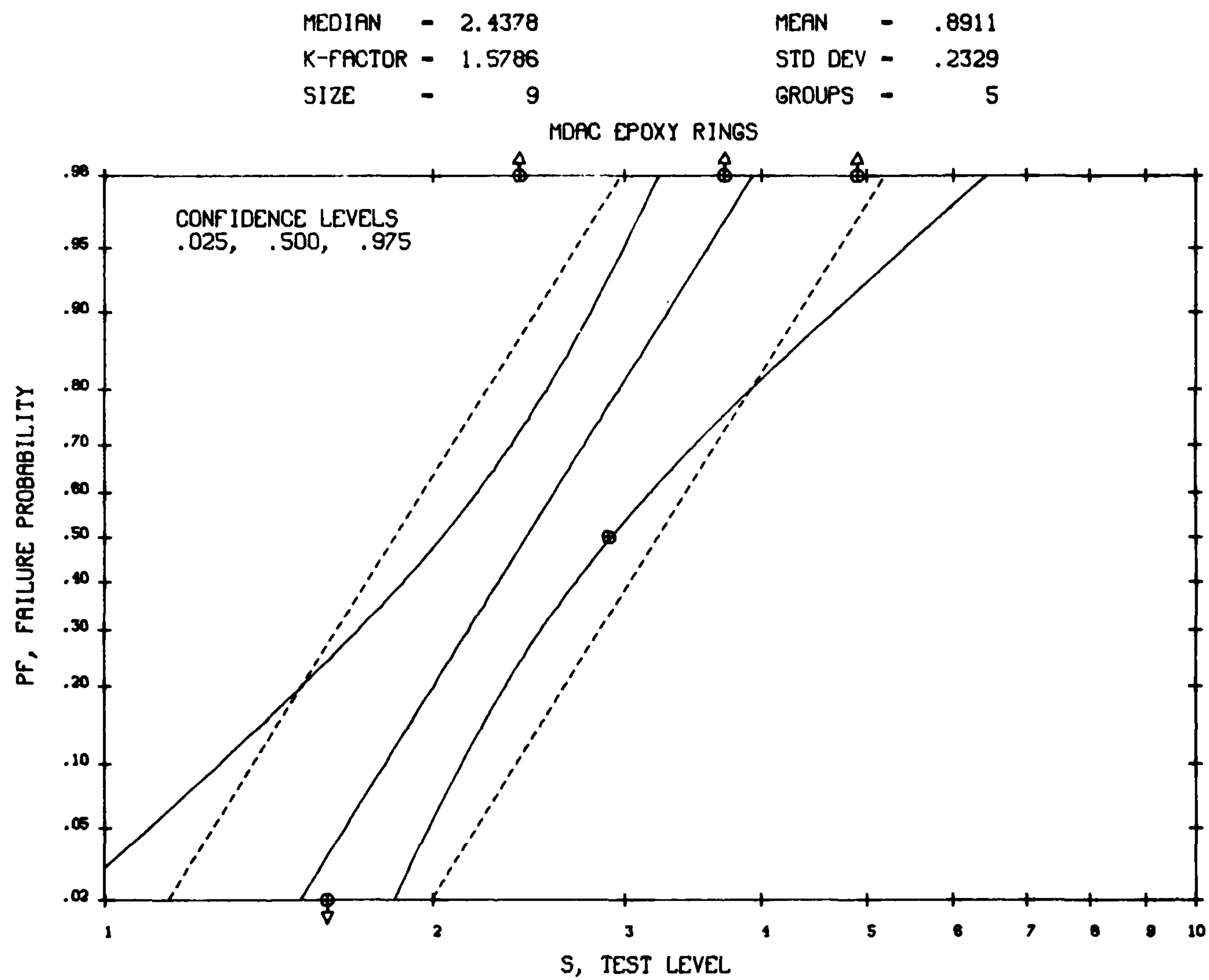


Figure 13. Transformed Plot of Constant K_B Matched at 20% P_F

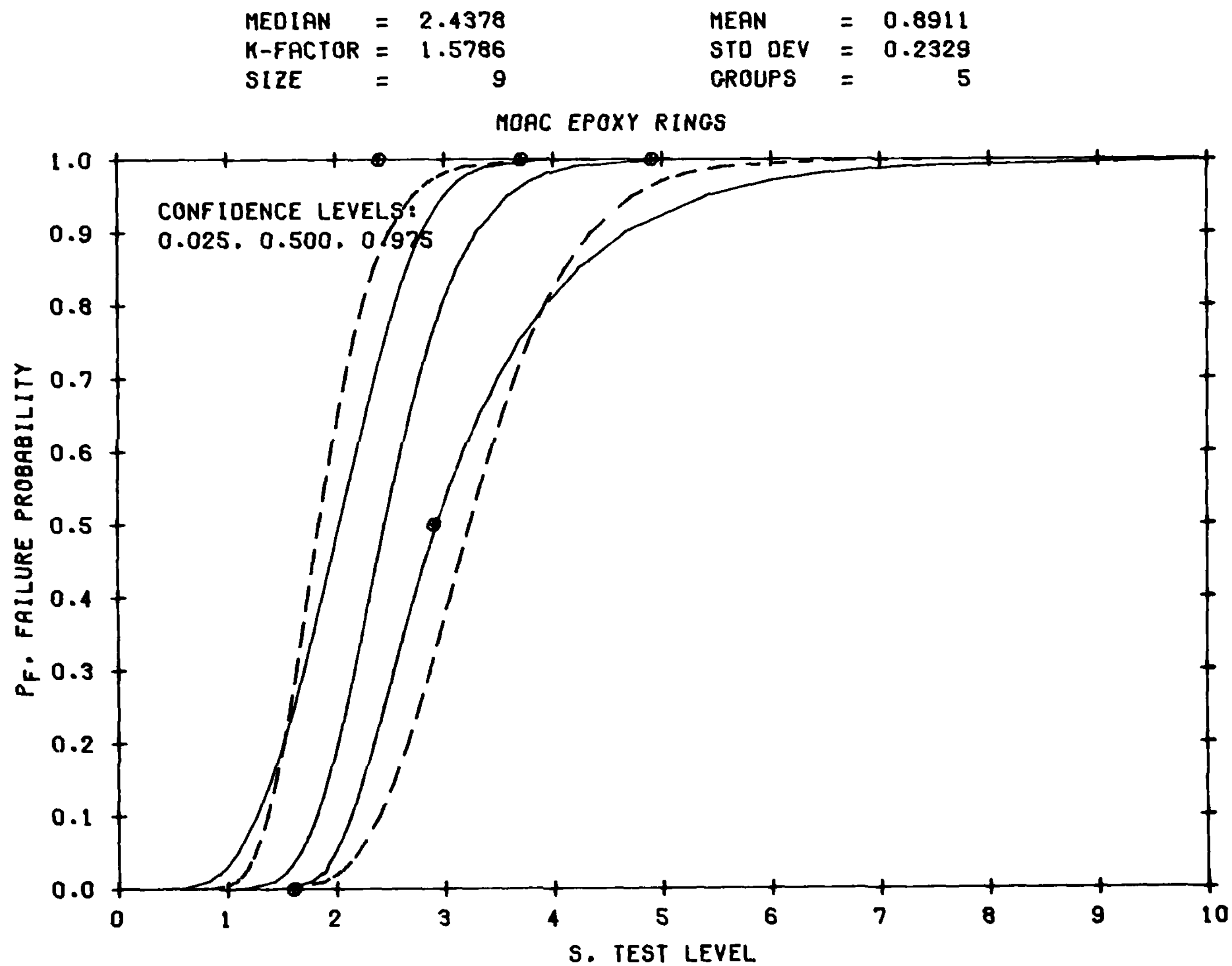


Figure 14. Natural Plot of Constant K_B Matched at 20% P_F

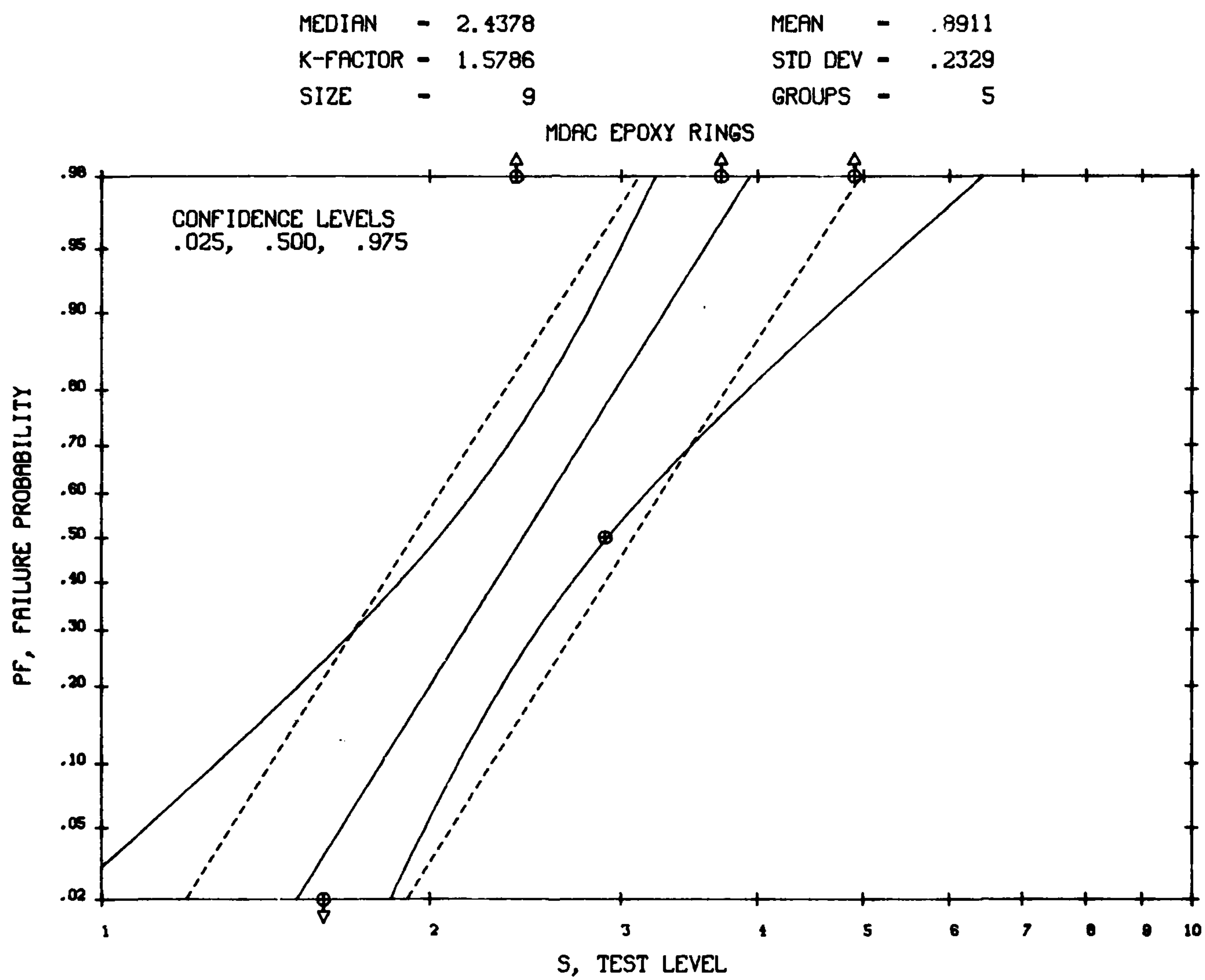


Figure 15. Transformed Plot of Constant K_B Matched at 30% P_F

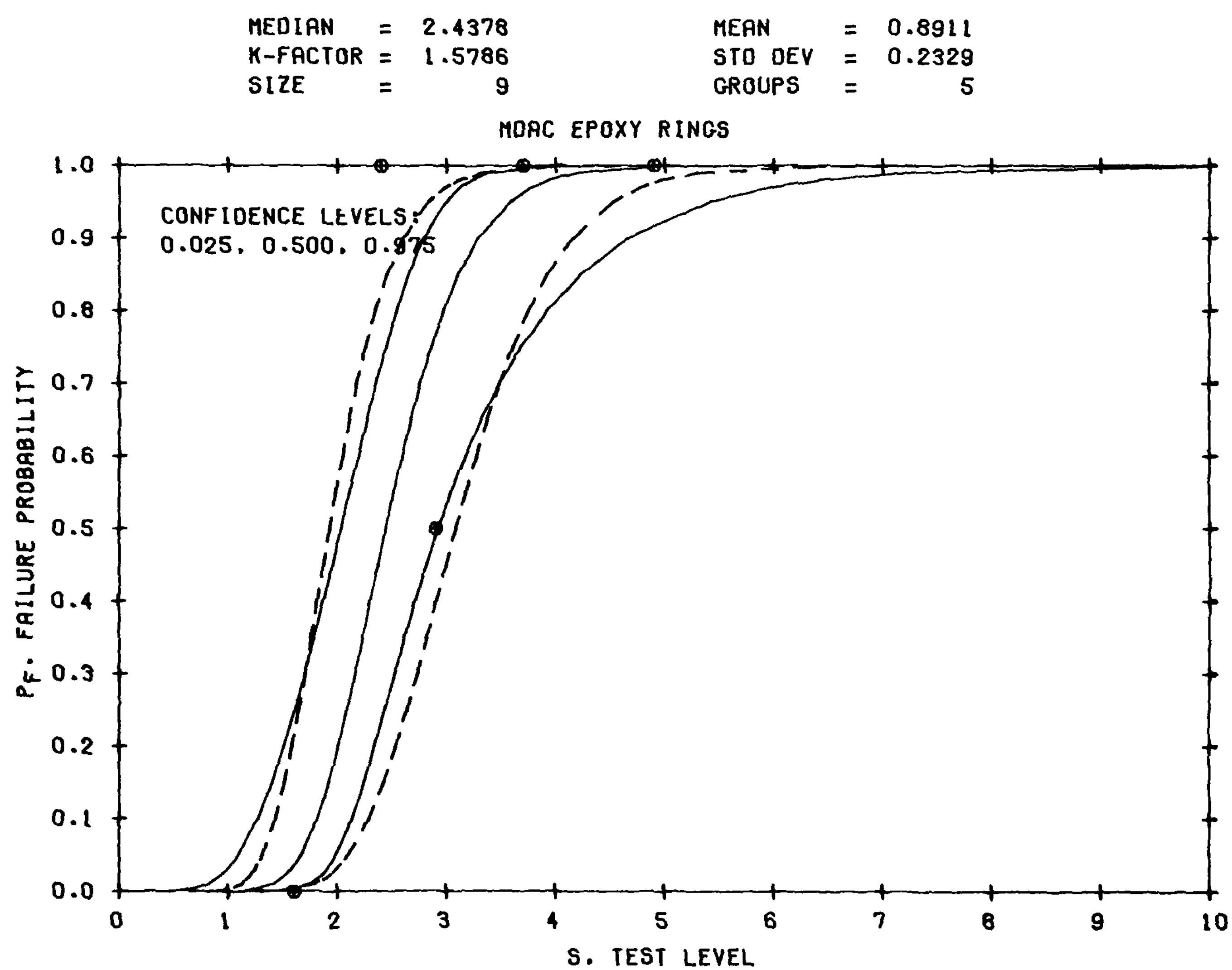


Figure 16. Natural Plot of Constant K_B Matched at 30% P_F

These elements of systematic uncertainty can be combined to give a total systematic uncertainty

$$K_B = \exp\left\{\left[(\log_e 1.32)^2 + (\log_e 1.20)^2 + (\log_e 1.10)^2\right]^{\frac{1}{2}}\right\} \quad (40)$$
$$= 1.41$$

This is the value of K_B for aero-shell spall used in the BASH code analyses of the reentry vehicle presented in Volume I.

4.0 CONDITIONAL PROBABILITY FAST

The FAST code has been applied under the NHEP program to assess the hardness of reentry vehicles to exoatmospheric X-rays and to determine the associated confidence level. Confidence, C, is defined as the estimated likelihood that at least a prescribed proportion, P, of a flight of reentry vehicles will have a given fluence capability, $\phi_{P/C}$. In running the FAST code for a given threat, responses are simulated at several fluence levels by Monte Carlo techniques. Confidence estimates $C_i(\phi_i, P)$ are calculated for each fluence at a prescribed probability. The fluence capability $\phi_{P/C}$. is then found by interpolation:

$$\phi_{P/C} = \phi_1 + \frac{(\phi_u - \phi_1)\,(C - C_1(\phi_1, P))}{C_u(\phi_u, P) - C_1(\phi_1, P)} \quad (41)$$

By repitition of this process, fluence capability can be plotted as a function of threat temperature, T. These ϕ-T plots are the desired end produce of the Nuclear Hardness Evaluation Procedures.

Early in the NHEP program, the question was raised of how the ϕ-T curve changes given one or more successful underground test of a reentry system, subsystem, or component. A partial answer to this question has already been obtained during a previous study.[2] In that study, the method of conjugate distributions[21] was used to update the survivability-versus-confidence curve at a given threat and fluence level, based upon r successful tests out of n. Assuming that P/C curves generated by the FAST code are adequately fitted by beta distribution functions with parameters r', n' then the post test distribution was estimated by a beta function having the parameters r" = r + r'; n" = n + n'.[21]

The method of conjugate distributions has a number of limitations which preclude its general use for updating ϕ-T plots. These limitations include:

1) Inability to correct for other than tested fluence levels;
2) Inability to correct for other than tested threats;
3) Inability to account for less than perfect simulation of the operational environment by the test environment;
4) Inability to update subsystem or systems survivability based on component testing; and

5) Questionable applicability of the beta distribution function for all systems and subsystems. Poor fits have occasionally been observed with this distribution.

However, these limitions have been overcome by a new version of FAST, known as Conditional Probability FAST, FAST7.

FAST7 computes conditional probability, defined as the probability that an event will occur given the condition that a different event is known to have occurred. The standard notation for the probability that event A will occur given that event B has occurred is $P(A|B)$.

Inputs to the previous versions of FAST are allowed in the form of series systems, such as $S1 = C1 \cap C2$, where a failure of either component C1 or C2 fails the system; or parallel systems, such as $S2 = C1 \cup C2$, where a failure of both components fails the system. Parentheses are also allowed in combination with the Boolean operators $\cap$ and $\cup$ to build fairly complex logic.

Parentheses and Boolean operators are not used in defining conditional probabilities. The only permitted forms for conditional probabilities are:

$$S3 = S1|S2, \tag{42}$$

or

$$S3 = S1|SS1 \tag{43}$$

or

$$S3 = S1|C1 \tag{44}$$

Systems or subsystems defined above may not be used further. However, these restrictions (discussed below) do not limit the general applicability of the method, since any series or parallel event can be represented as a system, subsystem, or component to designate a complex event.

After the conditional probability has been properly defined in the FAST7 input, the code computes the fluence capability by the following steps:

1) A set of bias variation values are randomly selected in the outer iteration loop;
2) The code computes a probability of survival for the systems or subsystems representing events A and B; and
3) The code locates the appropriate cell in the FAST output histogram for event A.

For example, if a probability of survival of 0.92 is computed, the appropriate cell might be represented by the cell boundaries 0.90 and 0.95, with the midpoint 0.925. Instead of adding a one to the sum in this cell,

the probability of survival of the system or subsystem representing event B is added to the sum. Then, a new set of systematic or bias variations are drawn in the outer iteration loop, and steps 1 through 4 are repeated.

At the end of a FAST7 run, the P/C histogram includes the estimated likelihoods, C", that the probability of survival of the system representing event A is at least some specified value, P, given that a system representing event B has survived. Hence, the confidence estimate, C", is the confidence in P_s (A) conditional upon survival of B. In fact, event B need not represent a successful test. A test failure could be modeled in the FAST code by using the reciprocal of the fragility parameter and the reciprocal of the fragility curve, as explained below.

Given conditional confidence estimates, C", fluence capability is computed as before

$$\phi_{P/C"} = \phi_1 + \frac{(\phi_u - \phi_1)\ (C" - C_1\ (\phi_1,\ P))}{Cu(\phi_u, P)\ -\ C_1\ (\phi_1, P)} \qquad (45)$$

Fluence-Threat capability plots are then obtained by repeating this process for other threats.

It is interesting to compare the method conjugate distributions discussed above with the method of FAST7. The method of conjugate distribution requires that both the test environment and the environment of the assessment be the same. The FAST7 method does not require this restriction because the system representing event B (the test outcome) can share some or all of the uncertainties of event A (the system assessment) without having the identical environment. Moreover, simulation uncertainties can be modeled in the FAST7 assessment by including an extra transfer function or USER subroutine with the component inputs which describe the tested failure modes. Furthermore, the models of the tested failure modes need not include all of the failure modes of the system A assessment model.

The FAST7 likelihood estimates do not rely upon beta distribution fits to subsystem and system survivability-confidence statements, and they can accurately predict the entire P/C curve provided sufficient Monte Carlo iterations are performed.

5.0 CONCLUSIONS

The purpose of this study was to develop and demonstrate new techniques for estimating the potential benefits of testing under uncertainty. A **methodology is presented for estimating the reduction in the systematic** uncertainty of a selected component resulting from testing, and for evaluating the system benefit associated with this uncertainty reduction.

Two different approaches for the analysis of the effects of testing were investigated during this study. The first is a method for analysis of test data that utilizes maximum likelihood estimation of the distributional parameters. This method is embodied in a new computer code named ELF, Evaluation of Likelihood Function. The second approach involves the use of conditional probability, and has been incorporated into a new version of the FAST code named FAST7. Both of these computer codes can be used to objectively evaluate systematic uncertainties based on test results.

As a result of the exploratory work performed with these computer codes during the present study, the following conclusions can be drawn:

- The ELF code, utilizing maximum likelihood estimation, was demonstrated to provide a simple and effective method to objectively measure the effects of testing on component uncertainty.
- The FAST7 code, utilizing conditional probability, was shown to provide a general technique for test analysis which can even account for uncertainties in test simulation.

6.0 REFERENCES

1. W. H. Rowan, et al., "Failure Analysis by Statistical Techniques (FAST)", DNA 3336F, TRW Systems Group, Redondo Beach, September 1974.

2. L. H. Donahue, J. J. Farrell, G. M. Teraoka, "Hardness Evaluation of Reentry Vehicles with FAST Methodology", TRW Defense and Space Systems Group, Redondo Beach, (Unpublished).

3. E. A. Fitzgerald, (Letter to D. J. Kohler, DNA), A3-200-ABCO-EAF-79-29, McDonnell Douglas Astronautics Company, Huntington Beach, January 22, 1979.

4. M. Abramowitz, I. A. Stegun, ed., Handbook of Mathematical Functions, NBS Applied Mathematics Series 55, National Bureau of Standards, U.S. Department of Commerce, Washington, D.C., May 1968.

5. J. Aitchison, J. A. C. Brown, The Lognormal Distribution, Cambridge University Press, Cambridge, 1957.

6. C. I. Bliss, "The Method of Probits", Science, 79, 38, (1934).

7. D. J. Finney, Probit Analysis, A Statistical Treatment of the Sigmoid Response Curve, Cambridge University Press, Cambridge, 1947.

8. M. G. Natrella, Experimental Statistics, Chapter 10, "Sensitivity Testing", Handbook 91, National Bureau of Standards, Washington, D.C., 1973.

9. "System/360 Scientific Subroutine Package, (360A-CM-03X) Version III, Programmer's Manual", H20-0205-3, International Business Machines Corporation, White Plains, N.Y., 1968.

10. P. G. Hoel, Introduction to Mathematical Statistics, Second Edition, John Wiley & Sons, Inc. New York, 1954.

11. S. S. Shapiro, M. B. Wilk, "An Analysis of Variance Test for Normality (Complete Samples)", Biometrika, 52, 591, (1965).

12. G. J. Hahn, S. S. Shapiro, Statistical Models in Engineering, John Wiley & Sons, Inc., New York, 1967.

13. N. L. Johnson, B. L. Welch, "Applications of the Non-Central t-Distribution", Biometrika, 31, 362-389 (1940).

14. D. B. Owen, "A Special Case of a Bivariate Non-Central t-Distribution", Biometrika, 52, 437-446, (1965)

15. D. B. Owen, "Tables for Computing Bivariate Normal Probabilities", Annals of Mathematical Statistics, 27, 1075-1090, (1956).

16. J. C. Young, C. E. Minder, "An Integral Useful in Calculating Non-Central t and Bivariate Normal Probabilities", Applied Statistics 23, 455-457, (1974)

17. G. J. Resnikoff, G. J. Lieberman, "Tables of the Non-Central t-Distribution", Stanford University Press, Stanford, 1957.

18. D. B. Owen, "Tables for Computing Bivariate Normal Probabilities", Annals of Mathematical Statistics, 27, 1075-1090, (1956).

19. E. L. Bombara, "Reliability of Compliance with One-Sided Specification Limits when Data is Normally Distributed", ARGMA-TR-2B1R, Army Rocket and Guided Missile Agency, U.S. Army Ordnance Missile Command, Huntsville, 15 September 1961.

20. E. S. Pearson, H. O. Hartley, ed., Biometrika Tables for Statisticians, Volume II, Section 14, "The Distribution of Non-Central t", Cambridge University Press, Cambridge, 1972.

21. H. Raiffa, R. Schlaifer, Applied Statistical Decision Theory, Harvard University, Boston, 1961

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